Towards a new model for Minimum Bias and the Underlying Event in Sherpa

Andy Pilkington - IPPP, Durham, and Manchester

Trento, March 2012

(on behalf of H Hoeth, F Krauss, V Khoze, A Martin, M Ryskin and K Zapp)

Introduction

- The old models: typical description of minimum bias and underlying event in MC generators
- Implementation of the KMR model in Sherpa
- Comparison to LHC data
- Outlook

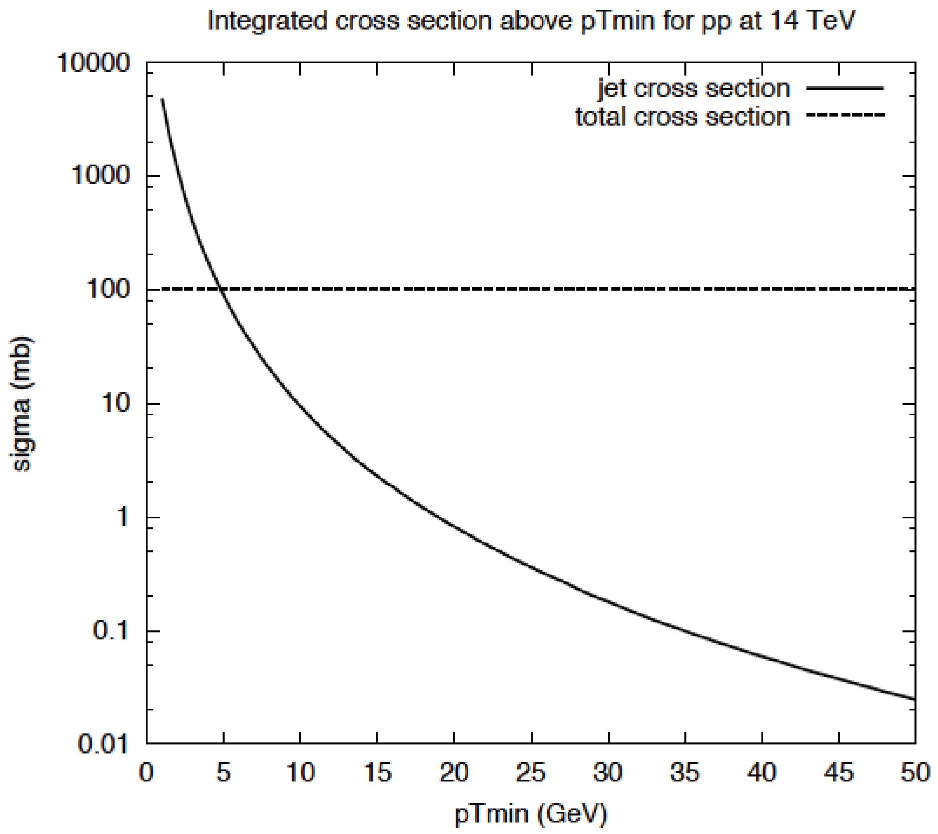
A typical model for multiple parton scattering (I)

- Interaction rate for QCD 2->2 scatters diverges as

$$\mathrm{d}\hat{\sigma}/\mathrm{d}p_{\perp}^{2} \approx 1/p_{\perp}^{4}$$

  and becomes larger than the total cross section as p_T tends towards zero.

- This is interpreted as the onset of multiple partonic scattering proton-proton interactions



$$\sigma_{\mathrm{int}}(p_{\perp\mathrm{min}}) = \iiint_{p_{\perp\mathrm{min}}} \mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}p_{\perp}^{2}\, f_1(x_1, p_{\perp}^{2})\, f_2(x_2, p_{\perp}^{2}) \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}p_{\perp}^{2}}$$

A typical model for multiple parton scattering (II)

$$\sigma_{\text{int}}(p_{\perp\text{min}}) = \iiint_{p_{\perp\text{min}}} \mathrm{d}x_1\, \mathrm{d}x_2\, \mathrm{d}p_\perp^2\, f_1(x_1, p_\perp^2)\, f_2(x_2, p_\perp^2) \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}p_\perp^2}$$

- The mean number of partonic interactions is given by

$$< n > = \frac{\sigma_{\text{int}}}{\sigma_{\text{nd}}} S(b)$$

  where S defines an overlap of proton matter distributions

- In each event, pick the number of (2->2) partonic interactions according to a Poisson distribution

  (but preserve energy-momentum conservation)

MPI model as a basis for UE and MB

- Underlying Event (UE):
  - e.g in Z events...
  - generate Z events according to the Z production cross section (LO, NLO, multi-leg.....)
  - generate additional 2->2 partonic scatters
  - pass each partonic scatter through parton shower algorithms, then the full set of partons through a hadronisation algorithm.
- Extension to Minimum Bias (MB)
  - Generate many 2->2 partonic scatters
  - pass each one through parton shower algorithms, then the full set of partons through a hadronisation algorithm.

Problems with this type of approach

- Diffraction is not included in this picture at all.

- For minimum bias events:
  - Pythia adds a diffractive process by hand, but no MPI between protons
  - Herwig++ ignores diffractive events

- For underlying event scatters:
  - The multiple scatters are always non-diffractive

- It would be nice to have a model that incorporates diffraction by default, for hard and soft processes......

Cross-sections in the eikonal approach

- Optical theorem links the elastic, inelastic and total cross-sections through an eikonal function Ω
- The eikonal function is related to the opacity of the proton

$$\sigma_{\rm tot}(s) = 2\int {\rm d}^2B_\perp \left\{1 - \exp\left[-\Omega(s,\, B_\perp)/2\right]\right\}$$

$$\sigma_{\rm inel}(s) = \int {\rm d}^2B_\perp \left\{1 - \exp\left[-\Omega(s,\, B_\perp)\right]\right\}$$

$$\sigma_{\rm el}(s) = \int {\rm d}^2B_\perp \left\{1 - \exp\left[-\Omega(s,\, B_\perp)/2\right]\right\}^2$$

Cross sections in multi-channel eikonal approach

- If the proton is expressed as a linear combination of orthogonal basis states (Good-Walker states)

$$|p\rangle = \sum_{i=1}^{S} a_i|\phi_i\rangle \qquad \langle\phi_i|\phi_k\rangle = \delta_{ik} \quad \text{and} \quad \sum_{i=1}^{S}|a_i|^2 = 1$$

then the eikonal is replaced by a combination of single channel eikonals, Ω_{ik}

$$\sigma^{pp}_{\rm tot} = 2\int d^2b_\perp \sum_{i,k=1}^{S}\left\{|a_i|^2|a_k|^2\left[1-e^{-\Omega_{ik}(b_\perp)}\right]\right\}$$

$$\sigma^{pp}_{\rm inel} = \int d^2b_\perp \sum_{i,k=1}^{S}\left\{|a_i|^2|a_k|^2\left[1-e^{-2\Omega_{ik}(b_\perp)}\right]\right\}$$

$$\sigma^{pp}_{\rm el} = \int d^2b_\perp \left\{\sum_{i,k=1}^{S}\left[|a_i|^2|a_k|^2\left(1-e^{-\Omega_{ik}(b_\perp)}\right)\right]\right\}^2$$

The KMR model

- The single-channel eikonals are defined as

$$\Omega_{ik}(Y, B_\perp) = \frac{1}{\beta_0^2} \int d^2 b_\perp^{(1)} d^2 b_\perp^{(2)} \delta^2 \left(\vec{B}_\perp - \vec{b}_\perp^{(1)} + \vec{b}_\perp^{(2)} \right) \Omega_{i(k)} \left(y, b_\perp^{(1)} \right) \Omega_{(i)k} \left(y, b_\perp^{(2)} \right)$$

- where each sub-term is obtained from the coupled evolution equations:

$$\frac{d\Omega_{i(k)}(y, b_\perp^{(1)}, b_\perp^{(2)})}{dy} = \exp\left\{ -\frac{\lambda}{2} \left[\Omega_{i(k)}(y, b_\perp^{(1)}, b_\perp^{(2)}) + \Omega_{(i)k}(y, b_\perp^{(1)}, b_\perp^{(2)}) \right] \right\} \cdot \Delta \cdot \Omega_{i(k)}(y, b_\perp^{(1)}, b_\perp^{(2)})$$

$$\frac{d\Omega_{(i)k}(y, b_\perp^{(1)}, b_\perp^{(2)})}{dy} = \exp\left\{ -\frac{\lambda}{2} \left[\Omega_{i(k)}(y, b_\perp^{(1)}, b_\perp^{(2)}) + \Omega_{(i)k}(y, b_\perp^{(1)}, b_\perp^{(2)}) \right] \right\} \cdot \Delta \cdot \Omega_{(i)k}(y, b_\perp^{(1)}, b_\perp^{(2)})$$

The KMR model (continued)

- The coupled evolution equations satisfy the following boundary conditions:

$$\Omega_{i(k)}(-Y/2,\, b_\perp^{(1)}) \ = F_i(b_\perp^{(1)})$$
$$\Omega_{(i)k}(+Y/2,\, b_\perp^{(2)}) \ = F_k(b_\perp^{(2)})$$

where the F's are form factors.

The KMR model implemented Sherpa

- Two channel eikonal used

$$|p\rangle = \frac{1}{\sqrt{2}}|\phi_1\rangle + \frac{1}{\sqrt{2}}|\phi_2\rangle\,.$$

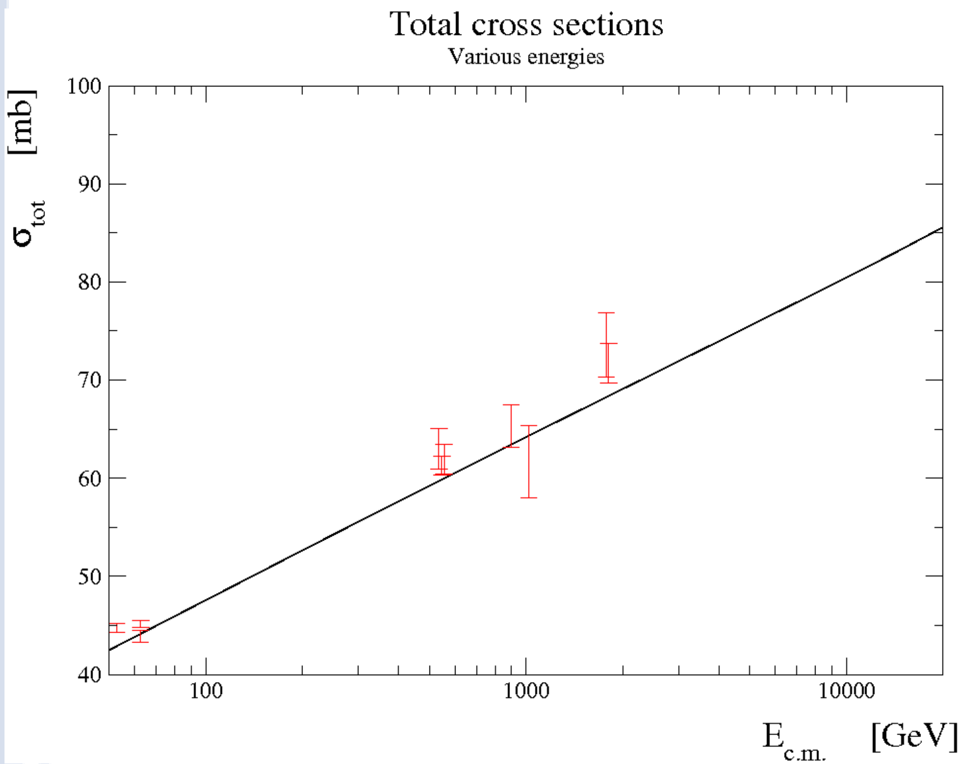
and the form factors (boundary conditions) are assumed to be dipole-like

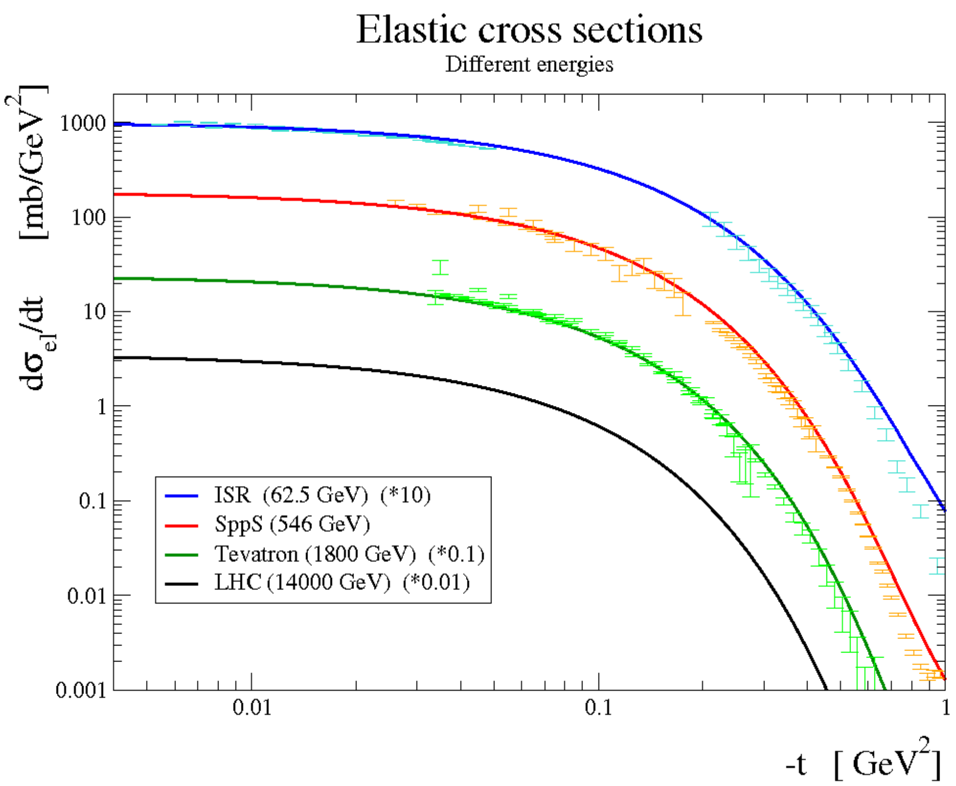
$$\mathcal{F}_{1,2}(q_\perp) \;=\; \beta_0^2\,(1\pm\kappa)\frac{\exp\left(-\frac{\xi(1\pm\kappa)q_\perp^2}{\Lambda^2}\right)}{\left(1+\frac{(1\pm\kappa)q_\perp^2}{\Lambda^2}\right)^2}$$

with $\Delta = 0.3$, $\lambda = 0.25$, $\beta_0^{\,2} = 30$ mb, $\kappa = 0.5$, $\Lambda^2 = 1.5$ GeV^2, $\xi = 0.225$

Inclusive results

- Total and elastic cross sections at various centre-of-mass energies





Event generation using new model in Sherpa (I)

- Select the mode (quasi-elastic, inelastic) according to cross sections (slide 8)

- If quasi-elastic is chosen
  - select elastic, low-mass SD or low-mass DD
  - select momentum transfer according to fourier transform
  - if diffractive, outgoing (diffractive) state is N*(1440)
  - event generation finished after decay of N*

Event generation using new model in Sherpa (II)

- If inelastic is chosen,
  - select Good-Walker state {i,k} according to cross section
  - select impact parameter (b) according to integrand.
- Choose number of partonic interactions (or 'ladders' in BFKL language) according to a Poisson distribution with mean set to Ω_{ik}

Initialising each ladders

- Select initial state flavour according to IR-continued PDFs
  - assume f(x,0) = valence only
  - keep norm of valence quarks, renormalise valence gluons to satisfy momentum sum rule
  - switch off sea with Q

- Weight by Regge-motivated cross-section $(s/s_0)^\eta$, where s_0 is fixed to reproduce inelastic cross section in this channel ik, $s>s_0$
  -

$$\hat{\sigma}_{ik} = \int_{\hat{s}_{\min}}^{\hat{s}_{\max}} \frac{\mathrm{d}\hat{s}}{2\hat{s}} \int_{-y_{\max}}^{y_{\max}} \mathrm{d}y \, \frac{x_+ F_1(x_+,0) x_- F_2(x_-,0)}{\hat{s}} \left(\frac{\hat{s}}{\hat{s}_{\min}} \right)^\eta$$

Generate ladder sub-structure

- t-channel exchange defined as singlet or octet using eikonals
  - $P_{\text{singlet}} = \{1-\exp[-\lambda\, (\Omega(y_1)-\Omega(y_2))/(2\Omega(y_1))]\}^2$
- If singlet, no emission allowed, i.e. rapidity gap, ladder complete.
- If octet, gluon emission generated according to pseudo-Sudakov:
  - Ordered in rapidity over allowed phase space

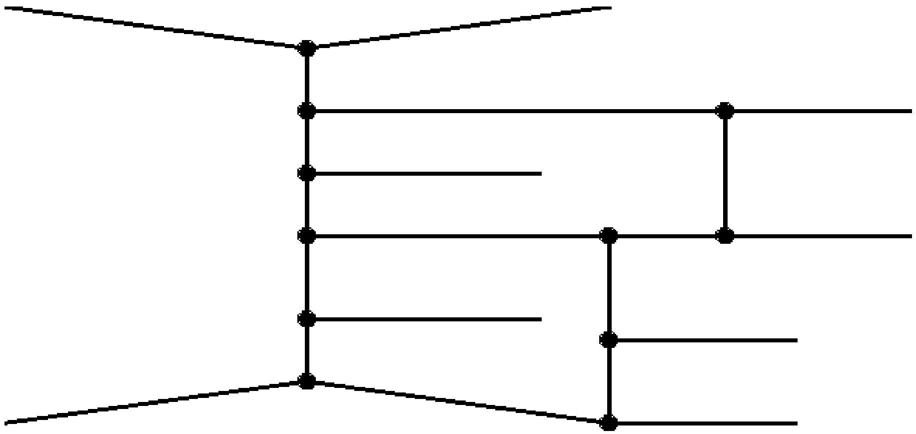
$$\exp\left\{-\int_{y_0}^{y_1} \mathrm{d}y \int \mathrm{d}k_\perp^2 \, \frac{C_\mathrm{A}\alpha_s(k_\perp^2)}{\pi} f(k_\perp^2) \min\left(1, \frac{k_0^2}{q^2}\right)^{\frac{C_\mathrm{A}\alpha_s(k_0^2)}{\pi}|y-y_0|} \exp\left[-\frac{\lambda}{2}\left(\Omega_{i(k)}(y)+\Omega_{(i)k}(y)\right)\right]\right\}$$

- These three steps are iterated. Emission ends when:
  - no more rapidity can be squeezed in, or
  - colour singlet configuration reached

Rescattering

- BFKL resums ladders within ladders, so need to allow rescattering between emitted gluons
  - Iterate over all pairs of partons
  - Construct rescattering probability for each pair as

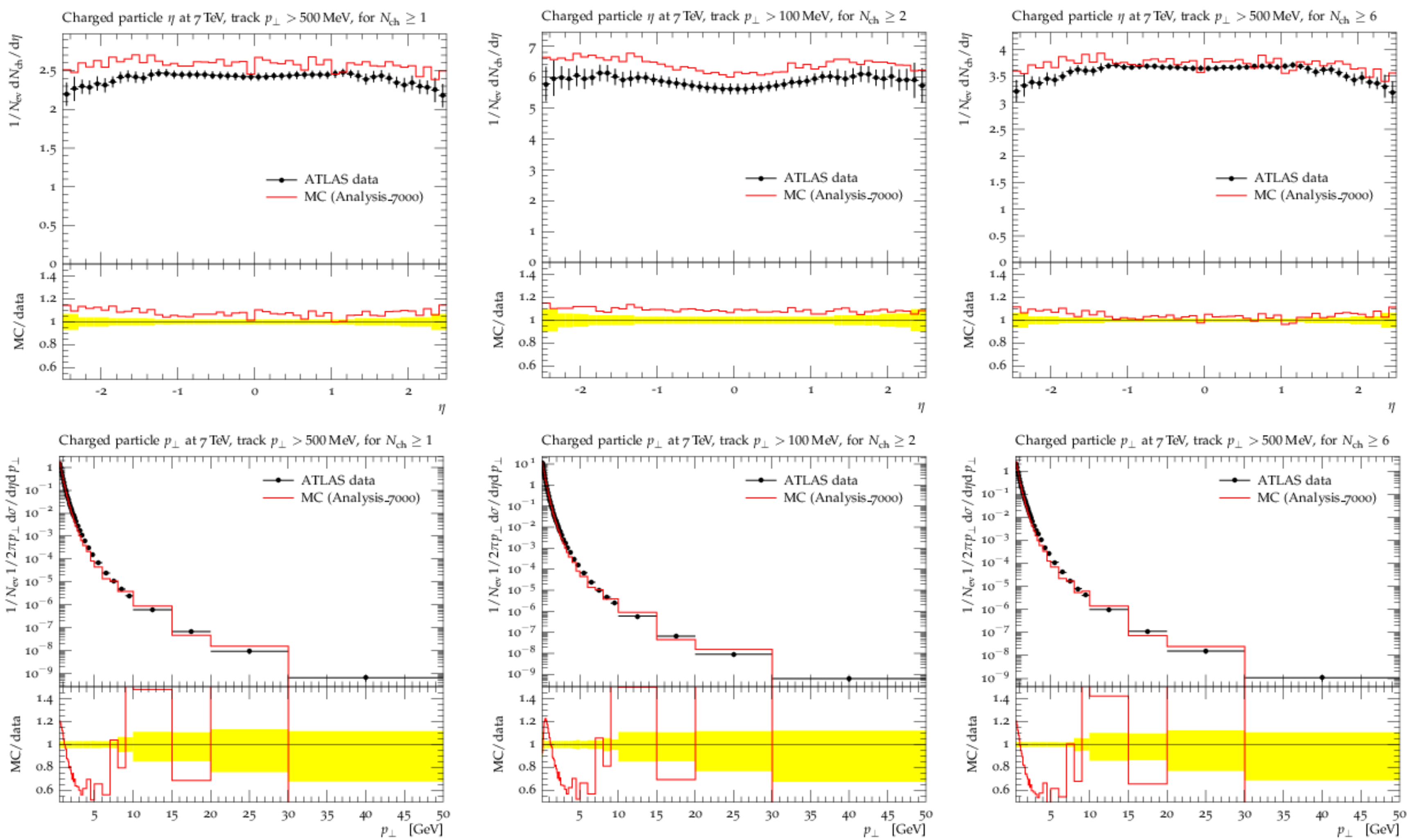
    $P_{rescatter} = 1\text{-}\exp\{-\lambda/2\ [\Omega(y_1)-\Omega(y_2))/(\Omega(y_1)]\}$
  - Check each pair (compare with random number)



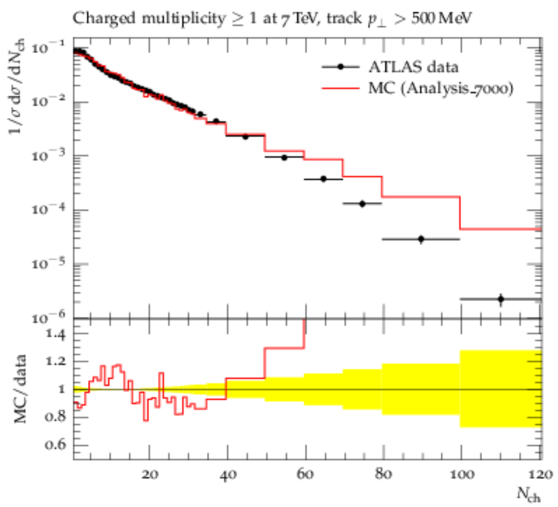
Untuned results

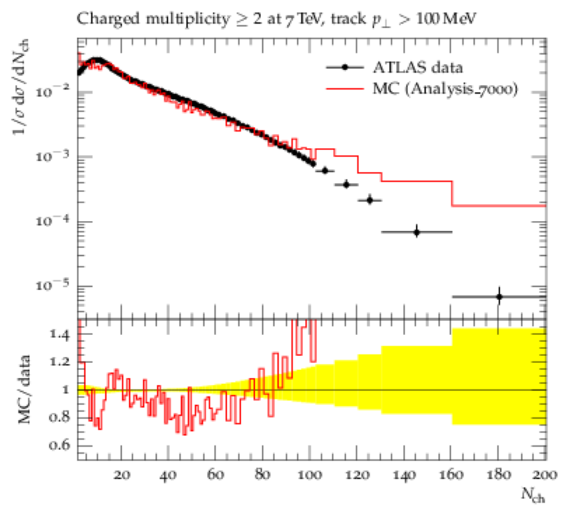
- Parameters of model:
  - Q_0 = 2 GeV, Δ = 0.4, λ = 0.5, Λ = 1.5 GeV, ξ = 0.225, κ= 0.55
- Inclusive quantities:
  - σ_{tot} = 99.2 mb, σ_{inel} = 68.5 mb, σ_{el} = 24.5 mb, σ_{SD} = 5.6 mb
  -
- 3.26 primary ladders/event, 5.2 in total (including rescattering)
- 22% of ladders have singlet, out of which 45% are hardest
- Colour reconnections switched off (can enable some if desirable)

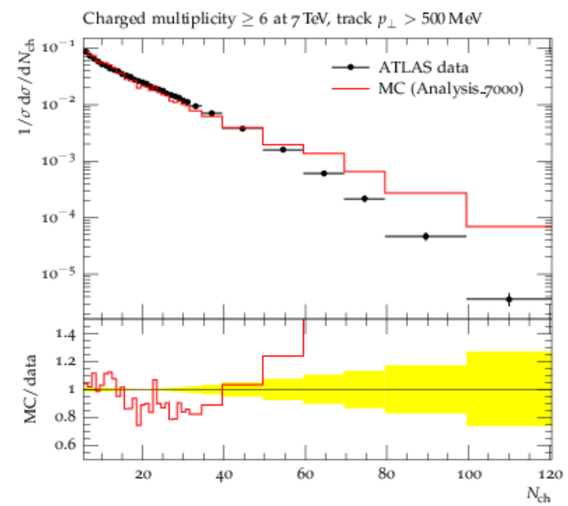
Comparison to LHC data (I)

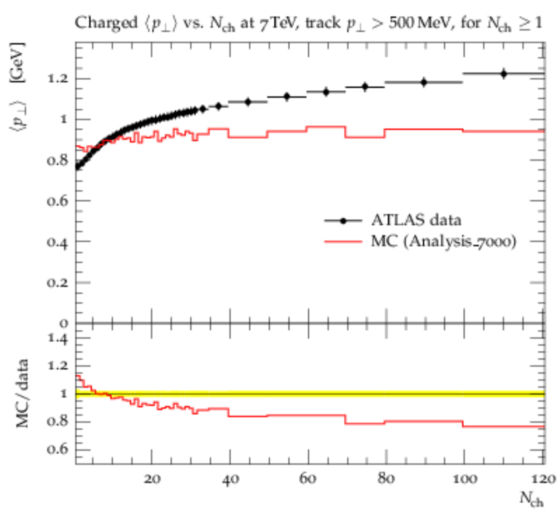


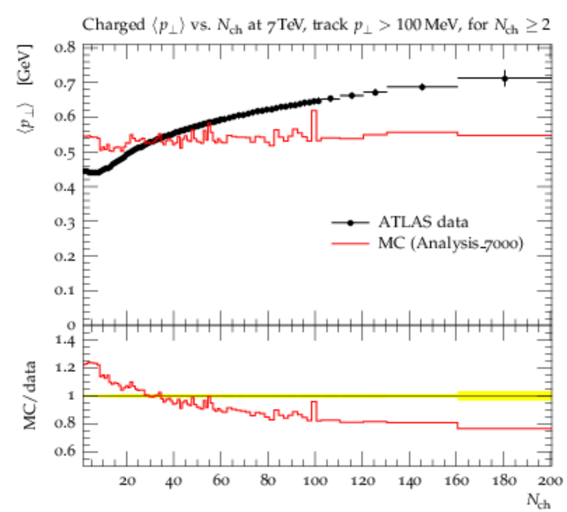
Comparison to LHC data (II)



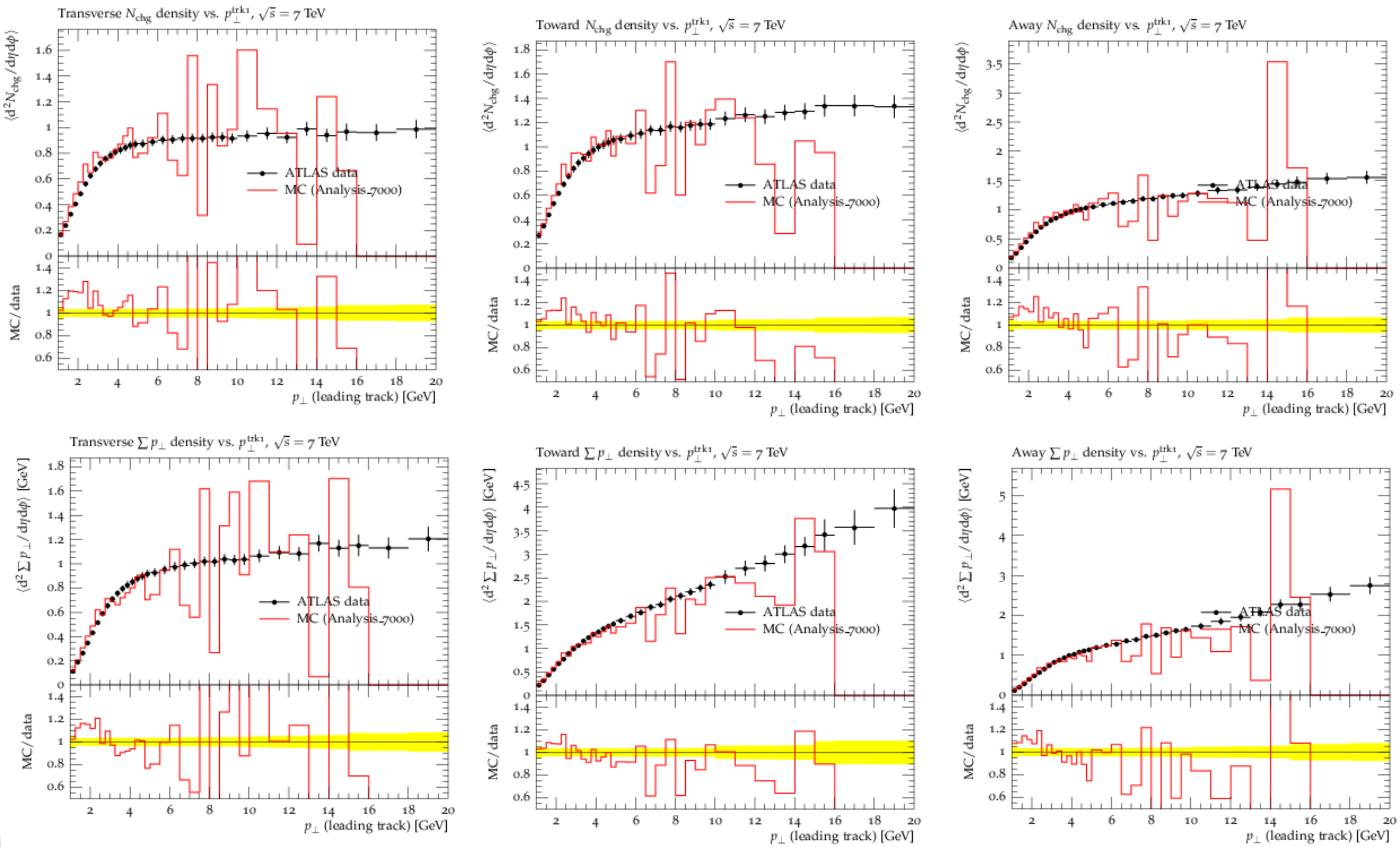




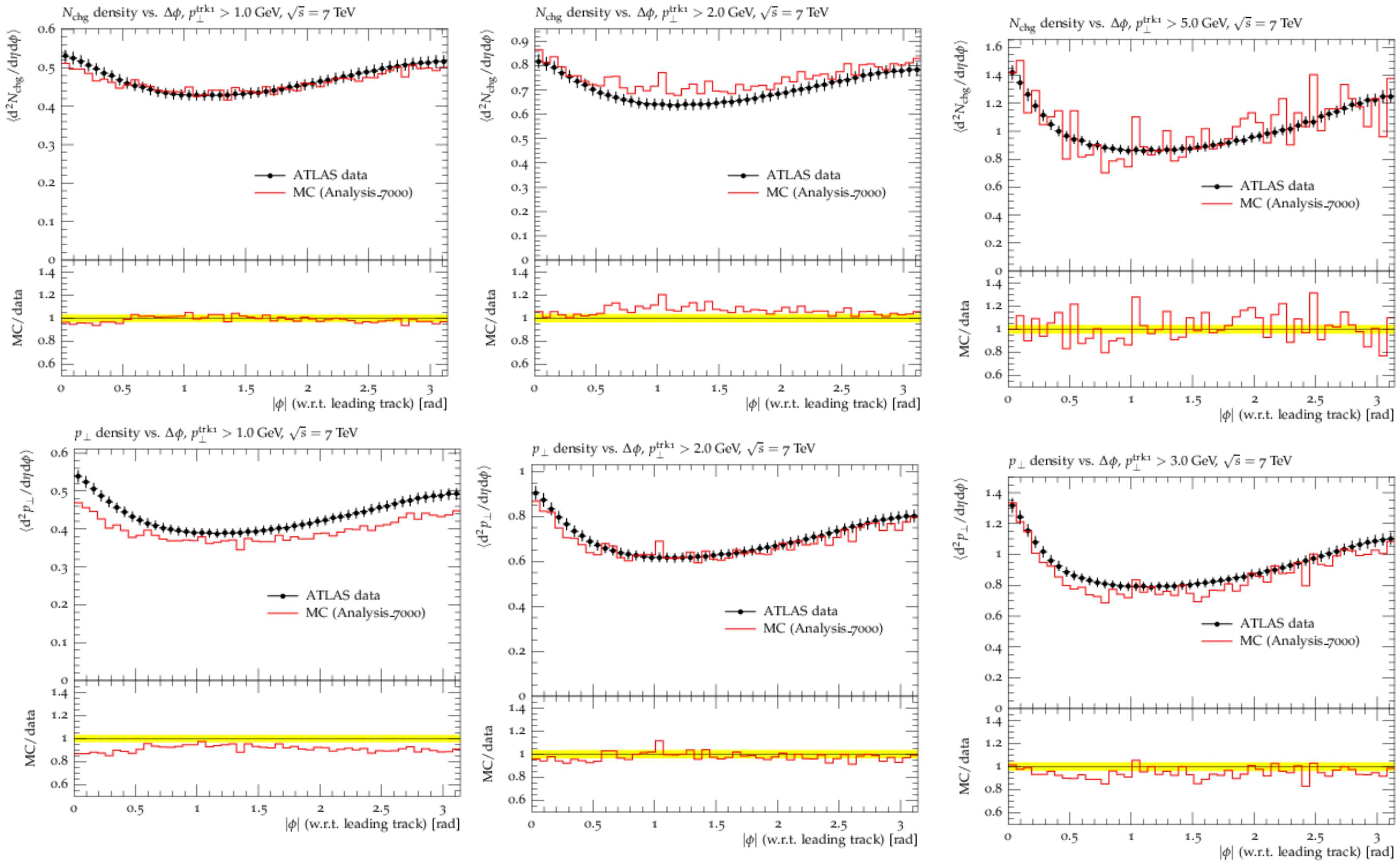




Comparison to LHC data (III)



Comparison to LHC data (IV)



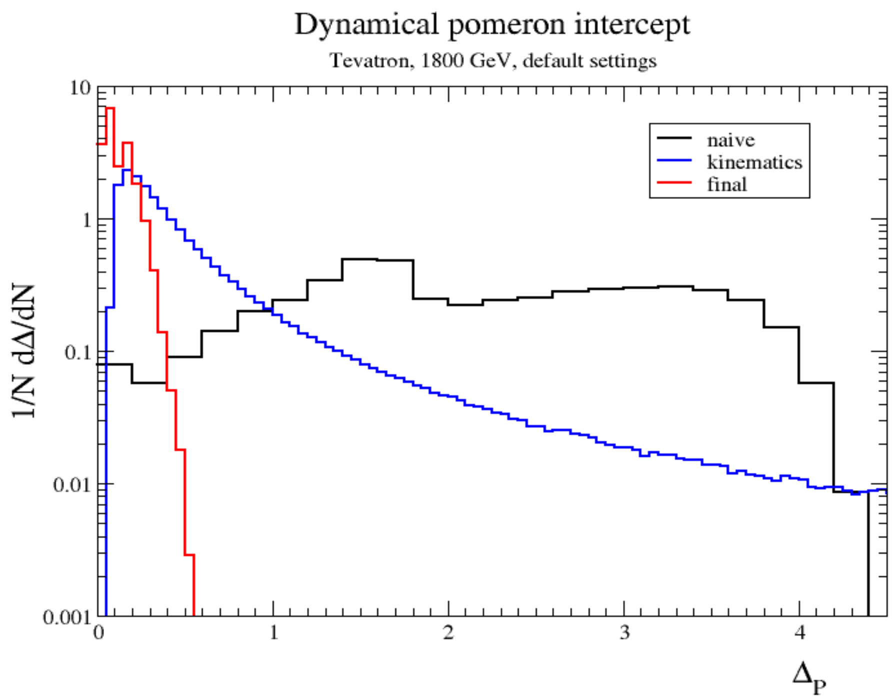
Summary and Outlook

- Implementation of the KMR model as a two-state model in the Sherpa event generator
  - Can add further states/reggeons (important for low energies)
  - Added dynamical picture of the gluon ladders
- Sherpa implementation is currently being tuned to latest LHC data
  - First release of model is imminent
  - Can use tuned output with string AND cluster hadronisation
- Still to come:
  - quarks in ladders, photons, mesons (quarkonia)
  - transformation into an underlying event model

Filling the ladders

- Generate emissions in between, according to "Sudakov form factor"

$$\mathcal{S}(y_0, y_1) = \exp\left\{ -\int_{y_0}^{y_1} \mathrm{d}y \int \mathrm{d}k_\perp^2 \frac{\alpha_s(k_\perp^2 + K_0^2)}{(k_\perp^2 + K_0^2)} \times \left[\frac{K_0^2}{q^2 + K_0^2}\right]^{\frac{3\alpha_s(q^2+K_0^2)}{\pi}|y-y_0|} \times \exp\left[-\frac{\lambda}{2}\left(\Omega_{i(k)}(y) + \Omega_{(i)k}(y)\right)\right] \right\}$$



- dynamical pomeron intercept
- Reweight ladder with ME ~ $1/t_{\text{hardest}}$ for hardest emission
- Note: At this point strictly t-channel, filling stops when either no more y can be "squeezed" in, or when "active" y-interval goes to singlet colour config.

Treatment of colours

($\rightarrow$ hard diffraction)

- In principle, BFKL equation resums “ladders in ladders”
- In as purely gluonic picture, this means that each t-channel propagator at LO is a (reggeised) gluon, but at all orders it is in a colour state given by something like C = [8] + [8]*[8] + [8]*[8]*[8] ...
- Take a look at the [8] * [8]: Its decomposition is something like

  [8] * [8] = [27] + [10] + [10] + [8] + [8] + [1],

  i.e. containing a singlet state.
- Will treat anything else as octet. Decision of whether singlet or octet based on eikonals $\rightarrow$ singlet if elastic scattering between two rapidities y_1 and y_2.

$$P_{singlet} = \{1-\exp[-\lambda\, (\Omega(y_1)-\Omega(y_2))/(2\Omega(y_1))]\}^2$$

IR-continued PDFs