# Exclusive wide-angle processes

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**Outline:**

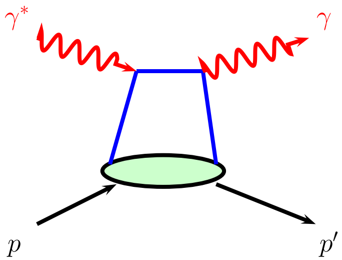
- **Factorization schemes**
- **Handbag factorization for** $\gamma\gamma \to M\overline{M}$
- **Flavor symmetry**
- **Results**
- **ERBL factorization for** $\gamma\gamma \to M\overline{M}$
- **Other exclusive wide-angle reactions**
- $\gamma\gamma \to B\overline{B}$
- **The crossed process - wide-angle Compton scattering**
- **Summary**

talk based on Diehl-Vogt-K hep-ph/0112274, Diehl-K 0911.3317

# Handbag factorization in excl. reactions

wide angles: large $s, -t, -u$ deeply virtual: large $Q^2$

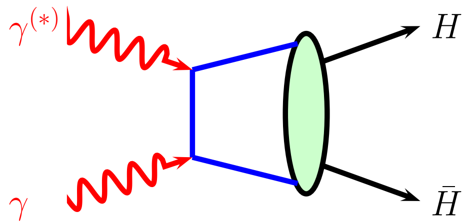
only one active parton (others are spectators, collinear fact.)




GPD

$\gamma^{(*)}p \to \gamma p$

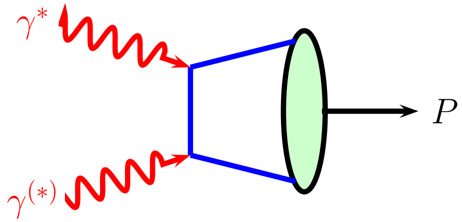
$\gamma^{(*)}p \to Mp$




2h-DA

$\gamma^{(*)}\gamma \to B\bar{B}, M\bar{M}$

$p\bar{p} \to \gamma^{(*)}\gamma, \gamma M$

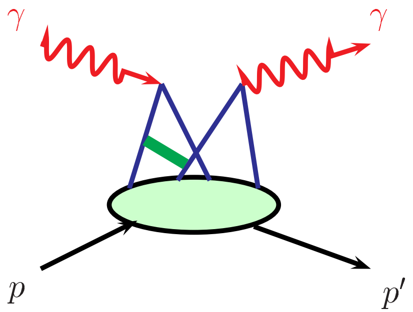



meson DA

$\gamma\pi(\eta, \eta')$ transition FF

other topologies (hard gluons required)

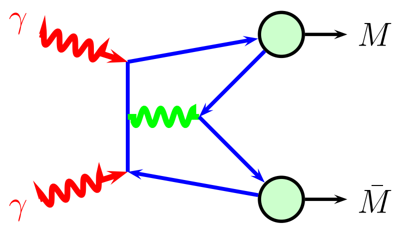
two (or more) active partons



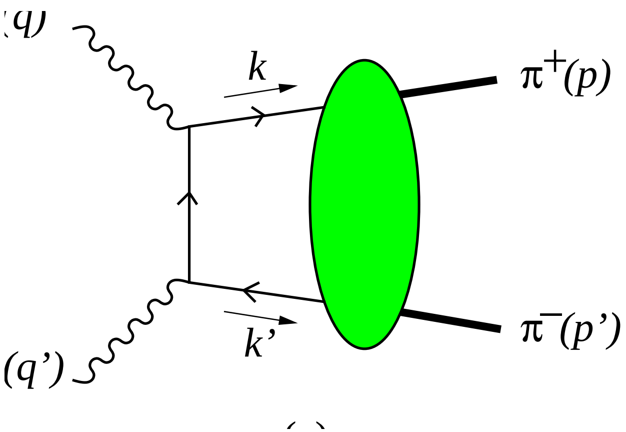

$\Longrightarrow$

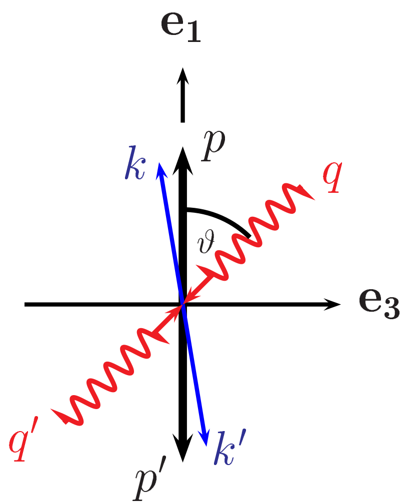
valence quark appr. ERBL fact.

e.g. $\gamma\gamma \to M\bar{M}$



# $\gamma\gamma \to M\bar{M}$




physical picture:

hard process $\gamma\gamma \leftrightarrow q\bar{q}$

soft $q\bar{q} \leftrightarrow h\bar{h}$ transitions

$s, -t, -u \gg \Lambda^2$

$\Lambda$ typical hadronic scale of order $1\,\mathrm{GeV}$

work in symmetric frame: $p^+ = p'^+$ $\qquad \zeta = \frac{p^+}{(p+p')^+} = 1/2 \qquad (v^\pm = (v_0 \pm v_3)/\sqrt{2})$

parton off-shell momenta: $k = \sqrt{\frac{s}{2}}[z, \bar{z} + \delta^-, \sqrt{2z\bar{z} + \delta_\perp}\mathbf{e}_\perp] \qquad z = \frac{k^+}{(p+p')^+}$

$k' = \sqrt{\frac{s}{2}}[\bar{z}, z - \delta^-, -\sqrt{2z\bar{z} + \delta_\perp}\mathbf{e}_\perp] \qquad \bar{z} = 1 - z$

to ensure soft $q\bar{q} \to \pi\pi$ transition: i) restricted transverse momenta $k_{\perp i}/z_i \sim \Lambda^2$

ii) all virtualities of the parton-hadron vertices are soft of order $\Lambda^2$

$\Longrightarrow 2z - 1, \sin\varphi, \delta^-, \delta_\perp \sim \Lambda^2/s$

$\varphi \simeq 0 : k \simeq p \quad k' \simeq p' \qquad \varphi \simeq \pi : k \simeq p' \quad k' \simeq p \qquad$ up to corr. of order $\Lambda^2/s$

after a lengthy calculation using light-cone techniques ($\mu, \mu'$ photon helicities)

$$A_{\mu\mu'} = -\sum_q (ee_q)^2 \int \frac{d^4k}{\sqrt{k^+k'^+}} \mathcal{H}_{\mu\mu'}(k,k')\, S(k,k') + \text{axial current term}$$

soft matrix element: ($\pi\pi$ production in $\gamma\gamma$ annihilation is C even)

$$S = \frac{1}{2}\int \frac{d^4x}{(2\pi)^4} e^{-ikx} \langle \frac{\pi^+(p)\pi^-(p') + \pi^+(p')\pi^-(p)}{2} \mid T\bar{q}(x)\gamma^+ q(0) \mid 0\rangle$$

charge conjugation invariance: $S(k,k') = -S(k',k)$ $\mathcal{H}(k,k') = -\mathcal{H}(k',k)$
two regions $k \simeq p$, $k \simeq p'$ are related through rotation by $\pi$ around 3-axis

Taylor expansion around 1-axis: ($\mathcal{H}_{\pm\pm} = 0$)

$$\mathcal{H}_{\pm\mp} = 2(\sqrt{u/t} - \sqrt{t/u}) - (z-\bar{z})(s/t + u/t) + \mathcal{O}((z-\bar{z})^2, \varphi^2)$$

$$\sim 1/\sin\theta \qquad\qquad \sim 1/\sin^2\theta$$

leading term vanishes due to a conspiracy of charge conj. inv. and rotation

$\gamma\gamma \to \pi\pi$ special situation

conspiracy does not occur in (and leading term of $\mathcal{H}$ dominates):

space-like region, e.g. real Compton scattering - $\gamma\pi(p) \to \gamma\pi(p)$
regions $k^+ > 0$ $(\gamma q \to \gamma q)$ and $k^+ < 0$ $(\gamma \bar{q} \to \gamma \bar{q})$
related by charge conj. but not by rotation

$\gamma\gamma \to B\bar{B}$: region $k' \simeq p$ corresponds to antiquark hadronization into $B$
requires sea quarks with very high momentum fractions
unlikely to exist

$z - \bar{z}$ term is parametrically of same order as parton-off-shellness effects
we remain with on-shell approximation
our approach to $\gamma\gamma \to \pi\pi$ is to be considered as a model

# $\pi\pi$ DA, form factors, amplitudes

$$\Phi^q_{2\pi}(z,\zeta=1/2,s) \;=\; \int \frac{dx^-}{2\pi} e^{-iz(p+p')^+x^-} \langle \pi^+(p)\pi^-(p')|\bar{q}(x)\gamma^+ q(0)|0\rangle\Big|_{x=[0,x^-,0_\perp]}$$

$$R^q_{2\pi} = \frac{1}{2}\int_0^1 dz(2z-1)\Phi^q_{\pi\bar{\pi}}(z,1/2,s) \qquad\qquad F^q_\pi = \int_0^1 dz \Phi^q_{2\pi}(z,1/2,s)$$

$$R_{2\pi} = e_u^2\, R^u_{2\pi} + e_d^2\, R^d_{2\pi} + e_s^2\, R^s_{2\pi}$$

$$A_{\pm\mp} = -\frac{16\pi\alpha_{\rm elm}}{\sin^2\theta} R_{2\pi}(s)$$

$$\frac{d\sigma}{dt}(\gamma\gamma\to\pi\bar{\pi}) = \frac{8\pi\alpha^2_{\rm elm}}{s^2}\,\frac{1}{\sin^4\theta}|R_{2\pi}(s)|^2$$

generalization to $\pi\to M$ straightforward

# SU(3) flavor symmetry

SU(2) symmetries associated with $u \leftrightarrow d$ Isospin $u(d) \leftrightarrow s$ V(U)-spin

photon behaves as $U$-spin singlet: – $M\overline{M}$ couples to $U = 0$

$$\Longrightarrow \qquad A_{K^+K^-} = A_{\pi^+\pi^-} \qquad\qquad A_{K^0\overline{K}^0} = \frac{3}{4}A_{\eta\eta} + \frac{1}{4}A_{\pi^0\pi^0} - \frac{\sqrt{3}}{2}A_{\eta\pi^0}$$

only $q\bar{q}$ intermediate state: – absence of $I = 2$ and $V = 2$

$$\Longrightarrow \qquad A_{\pi^0\pi^0} = A_{\pi^+\pi^-} \qquad\qquad A_{K^+K^-} = \frac{3}{4}A_{\eta\eta} + \frac{1}{4}A_{\pi^0\pi^0} + \frac{\sqrt{3}}{2}A_{\eta\pi^0}$$

number of inequalities and relations among cross sections, e.g.

$$2\frac{d\sigma_{\eta\pi^0}}{dt} + 3\frac{d\sigma_{\eta\eta}}{dt} = \frac{d\sigma_{K^+K^-}}{dt} + 2\frac{d\sigma_{K^0\overline{K}^0}}{dt}$$

and many relations among the form factors

only two independent: $R^u_{2\pi}$, $R^s_{2\pi}$

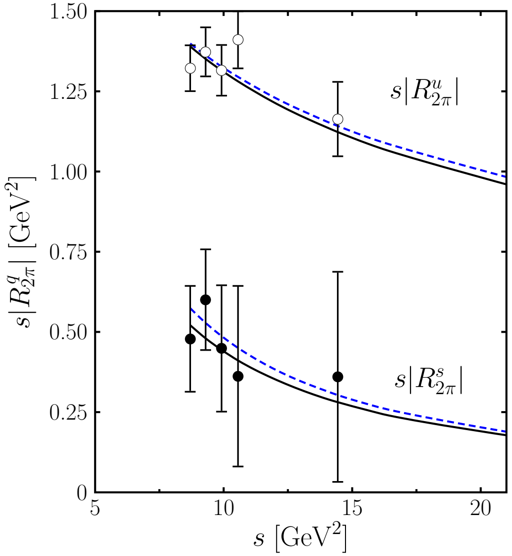
$$R_{\pi^+\pi^-} = R_{\pi^0\pi^0} = R_{K^+K^-} = \tfrac{5}{9} R^u_{2\pi} + \tfrac{1}{9} R^s_{2\pi}$$ (smallest contr. from non-valence)

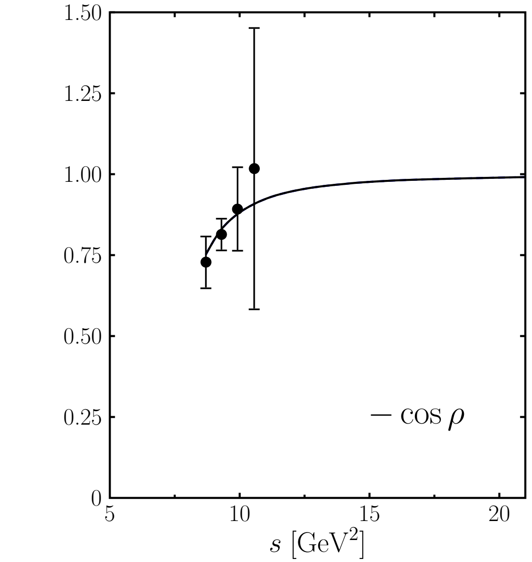
$$R_{K^0\overline{K}^0} = \tfrac{2}{9} R^u_{2\pi} + \tfrac{4}{9} R^s_{2\pi}$$ (strongest contr. from non-valence)

$$R_{\eta\pi^0} = \tfrac{1}{3\sqrt{3}} (R^u_{2\pi} - R^s_{2\pi}), \qquad R_{\eta\eta} = \tfrac{1}{3} (R^u_{2\pi} + R^s_{2\pi})$$

$R^u_{2\pi}$, $R^s_{2\pi}$ and rel. phase from cross sections on $K^0\overline{K}^0$, $\eta\pi^0$, $K^+K^-$

# Extraction of form factors

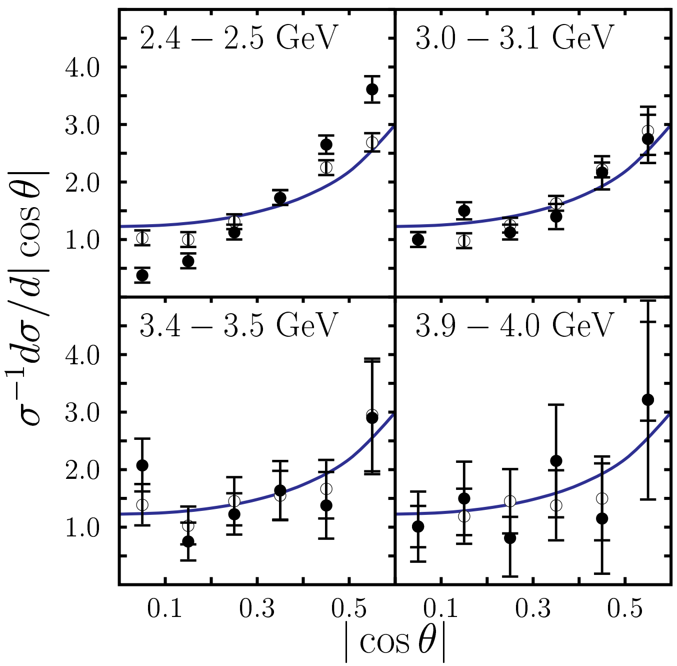



$s|F_\pi^{elm}| = 0.93 \pm 0.12\mathrm{GeV}^2$

**BELLE**

$\gamma\gamma \to \pi^+\pi^-(\bullet), K^+K^-(\circ)$




$\propto 1/\sin^4\theta$ (for $s \simeq 9\,\mathrm{GeV}^2$)

$s|R^q_{2\pi}| = a_q \left(\frac{s_0}{s}\right)^{n_q}$

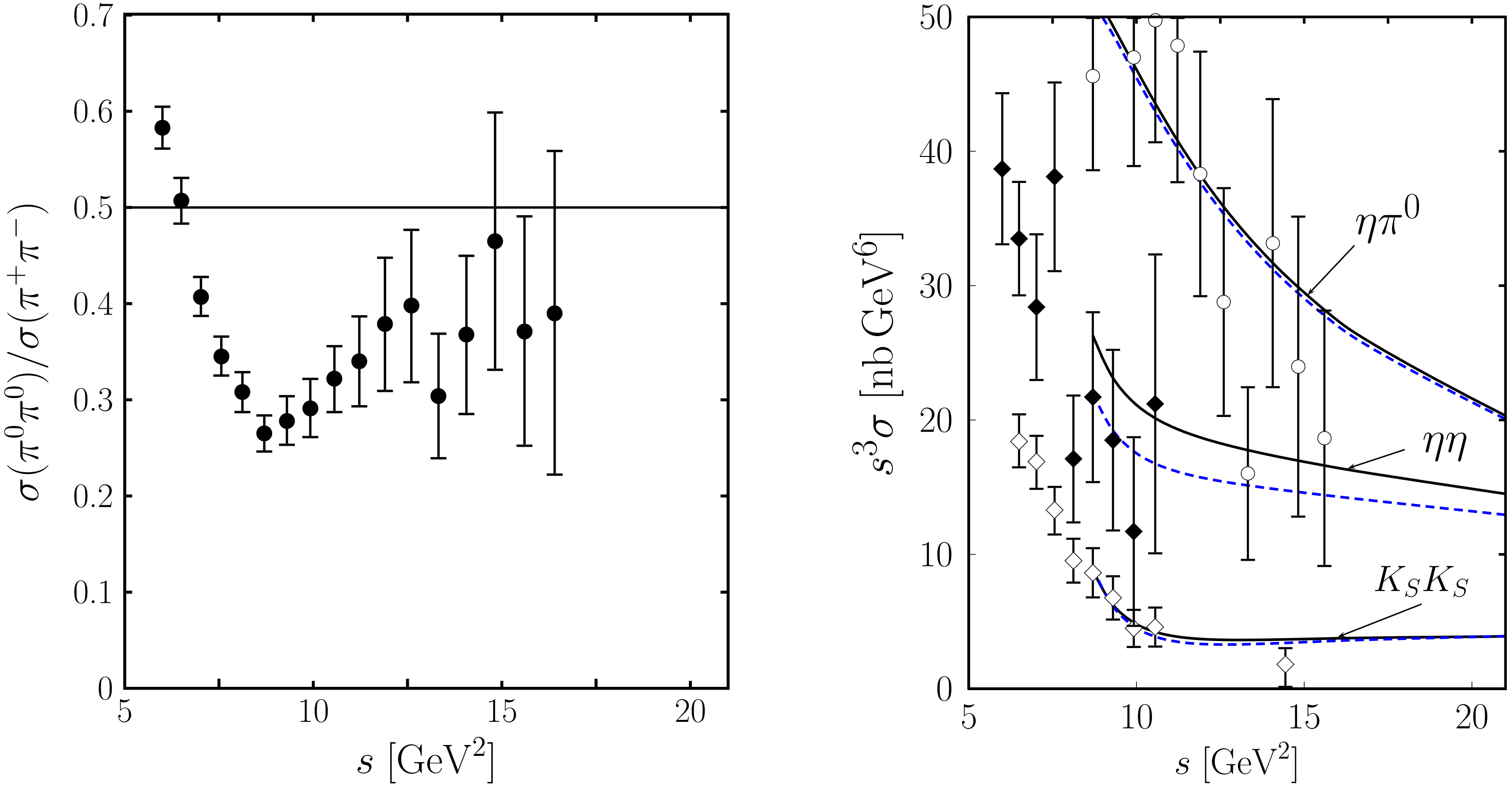
direct extraction ($K^0\overline{K}^0$, $\eta\pi^0$, $K^+K^-$):

$a_u = 1.37\,\mathrm{GeV}^2$, $n_u = 0.42$

$a_s = 0.50\,\mathrm{GeV}^2$, $n_s = 1.22$

relative phase close to $180°$

# Results on cross sections




$-0.6 < \cos\theta < 0.6$ data: BELLE

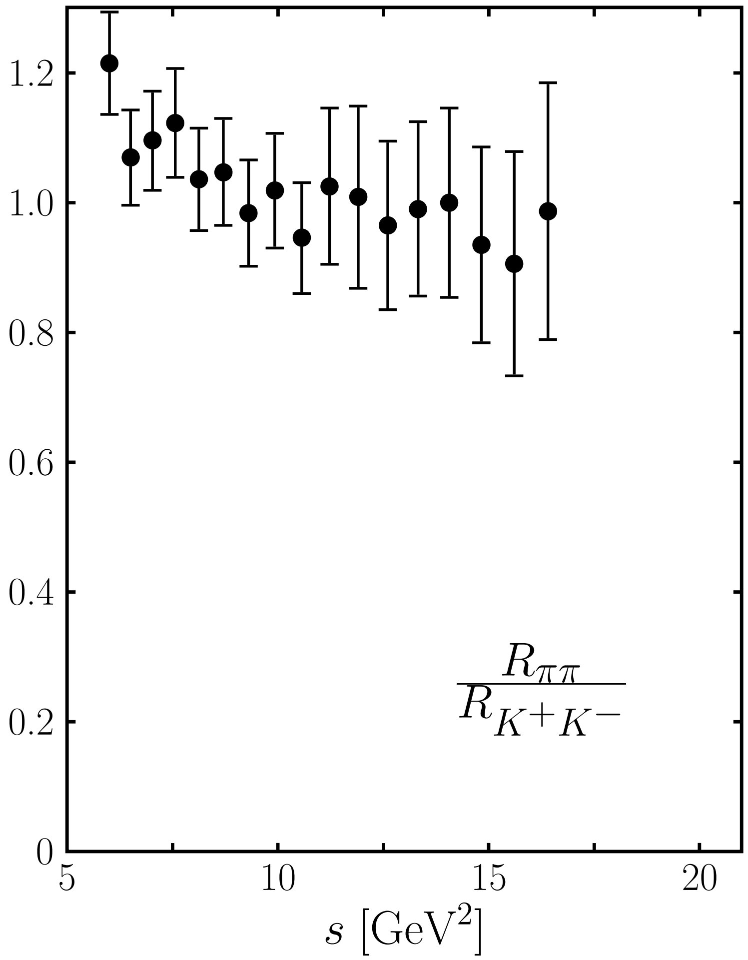
$A_{\pi^=\pi^-} = (\sqrt{2}A_{2\pi}^{I=0} + A_{2\pi}^{I=2})/\sqrt{6}$ $\eta = \eta_8$ (solid)

$A_{\pi^0\pi^0} = (A_{2\pi}^{I=0} - \sqrt{2}A_{2\pi}^{I=2})/\sqrt{3}$ with mixing (dashed)

consistent with dominance of $I = 0$ for large $s$

with little admixture of $I = 2$ (if in phase $A_2/A_0 < 0.11$)

# pion-kaon comparison




$R_{\pi\pi}$ extracted from

$$\frac{2}{3}\left[\sigma(\gamma\gamma \to \pi^0\pi^0) + \sigma(\gamma\gamma \to \pi^+\pi^-)\right]$$

interference term between $I = 0$ and $I = 2$ cancels

extracted form factor only mildly contaminated by $I = 2$

ratio compatible with 1 at large $s$

# ERBL factorization

requires very large scales in order to hold (dramatic failures in normalizations of $\pi$ and proton FF, Compton scattering, $\gamma\gamma \to M\overline{M}, p\overline{p}$)

$\gamma\gamma \to M\overline{M}$: $A \sim f_{M_1} f_{M_2} \int dx \Phi_{M_1}(x) \int dy \Phi_{M_2}(y) T(x, y, \theta)$
charged mesons $d\sigma/dt \propto 1/\sin(\theta)^4$, $\sigma \propto s^{-3}$ BELLE $n \simeq 4$

$$\sigma(M_1\overline{M}_1)/\sigma(M_2\overline{M}_2) \propto (f_1/f_2)^4$$

$(f_K/f_\pi)^4 \simeq 2$ requires $SU(3)_F$ breaking in distr. ampl. (higher Gegenbauer coeff) - parameters!

production of neutral mesons suppressed - bulk of ampl. $\sim (e_{q_1} - e_{q_2})^2$
explicite calculations (Brodsky-Lepage(80), Benayoun-Chernyak(90)):
$\pi^0\pi^0/\pi^+\pi^- \lesssim 0.05$ ($I = 0$ and $I = 2$ of comparable size)

according to Chernyak(06,09):
another dyn. mechanism required for neutral mesons:

like handbag but with only one quark between meson DAs plus additional gluon
light-cone sum rule estimate of principal behavior
$\sigma \propto s^{-5}$ (valid for highly asymmetric behavior?)

| | | | |
|---|---|---|---|
| BELLE | $\pi^0\pi^0$, $\eta\eta$: | $n \simeq 4$ | |
| | $K_s^0 K_s^0$, $\pi^0\eta$: | $n \simeq 5$ | errors of order 1 |

Actual fits to all data do not exist as yet

# More reactions

$p\overline{p} \leftrightarrow \gamma^*\gamma$, $\gamma^*\gamma \to M\overline{M}$ Pire et al (98)

$Q^2(s)$ is large scale, holds for all angles

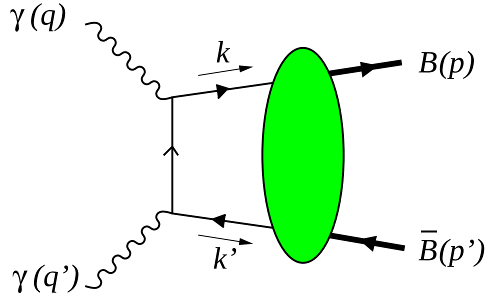
no data

$p\overline{p} \leftrightarrow \gamma\gamma$, $\gamma\gamma \to B\overline{B}$

Diehl et al (98), Weiss et al (98)

$B\overline{B}$-DAs, data from BELLE, CLEO, L3

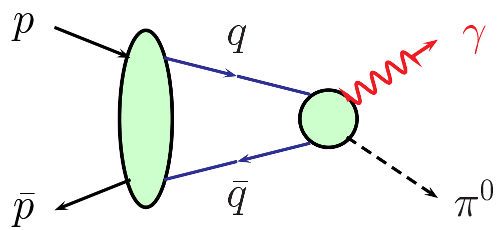
more from PANDA?




$p\overline{p} \to \gamma\pi^0(M)$

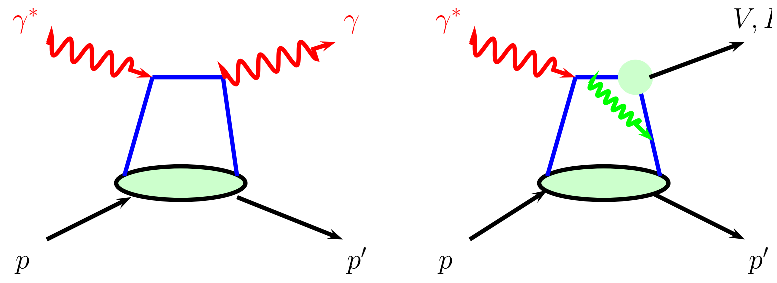
K.-Schäfer(05)

$p\overline{p}$-DAs, data E665, more from PANDA?

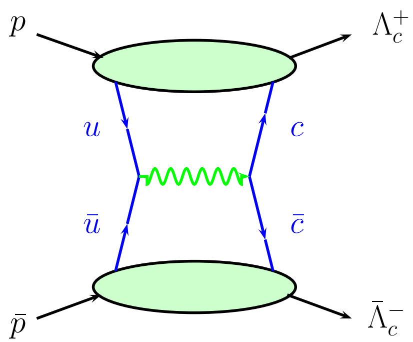


# Space-like reactions

Compton scattering and
meson production
Diehl et al(98), Radyushkin (98)
large $s, -t, -u$, $\quad p \to p$ GPDs
data from Jlab




$p\overline{p} \to \Lambda_c \overline{\Lambda}_c$
Goritschnig et al (09)
large scale $m_c$, $\quad p \to \Lambda_c$ GPDs
no data



# $\gamma\gamma \to p\overline{p}$

arguments for factorization as for mesons

4 2h-DAs $\Phi_i^q(z,\zeta,s) \quad i = V, A, P, S$

sum rules:

$F_i = \int_0^1 dz \Phi_i^q(z,\zeta,s) \qquad i = V, A, P \qquad (2\zeta - 1)F_S^q = \int_0^1 dz \Phi_S^q(z,\zeta,s)$

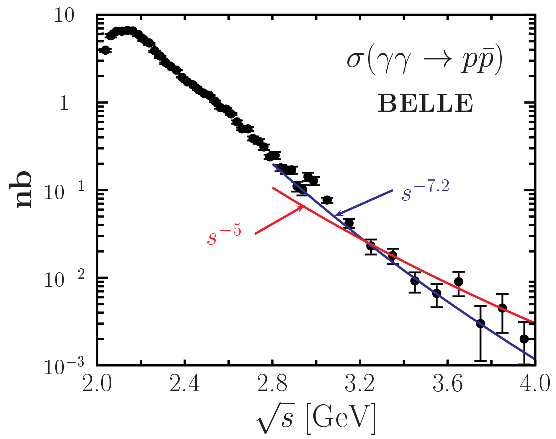
$G_M^p = \sum e_q F_V^q$ (magnetic) $\qquad F_2 = \sum e_q F_S^q$ (Pauli)

$$\frac{d\sigma}{dt}(\gamma\gamma \leftrightarrow p\,\bar{p}\,) = \frac{4\pi\alpha_{\mathrm{elm}}^2}{s^2 \sin^2\theta} \Big\{ \big| R_A(s) + R_P(s) \big|^2 + \frac{s}{4m^2} \big| R_P(s) \big|^2 + \cos^2\theta \, \big| R_V(s) \big|^2 \Big\}$$

$$R_i(s) = \sum_q e_q^2 F_i^q \qquad \mathrm{i = V,\ A,\ P}$$

Diehl, K., Vogt, hep-ph/0206288

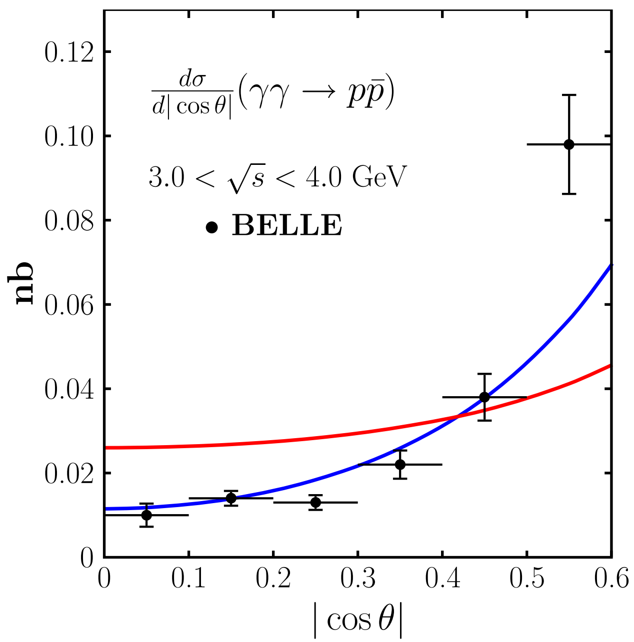
# $\gamma\gamma \to p\bar{p}$




$|\cos\theta| \leq 0.6$ BELLE hep-ex/0503006

$s^2 |G_M| \simeq 3\,\mathrm{GeV}^4$

$R_{\rm eff} = \sqrt{|R_A + R_P|^2 + \frac{s}{4m^2}|R_P|^2}$




$s^2 R_{\rm eff} = 2.9\,\mathrm{GeV}^4 (\frac{s}{10.4})^{-1.1}$

$s^2 R_V = 8.2\,\mathrm{GeV}^4 (\frac{s}{10.4})^{-1.1}$

$R_P$ from $p\bar{p}$ helicity correlation

PANDA: test of factorization?

$\gamma\gamma \to q\bar{q}$ for point-like fermions:

$|R_V^p| = |R_A^p|; \quad R_P^p = 0: \quad \frac{d\hat{\sigma}}{dt} \propto \frac{1+\cos^2\theta}{\sin^2\theta}$

# Flavor Symmetry

analogously to meson case: ($B$ ground state baryon)

absence of $I = 2$ final states, relations due to isospin and $U$-spin

FF for all $B\overline{B}$ channels described by three independent FFs $F^p_{iu}$, $F^p_{id}$, $F^p_{is}$

simplify: $\rho_d = F^p_{id}/F^p_{iu}$ $\rho_s = F^p_{is}/F^p_{iu}$

$\Longrightarrow \sigma(\gamma\gamma \to B\overline{B}) = r_B^2 \sigma(\gamma\gamma \to p\overline{p})$

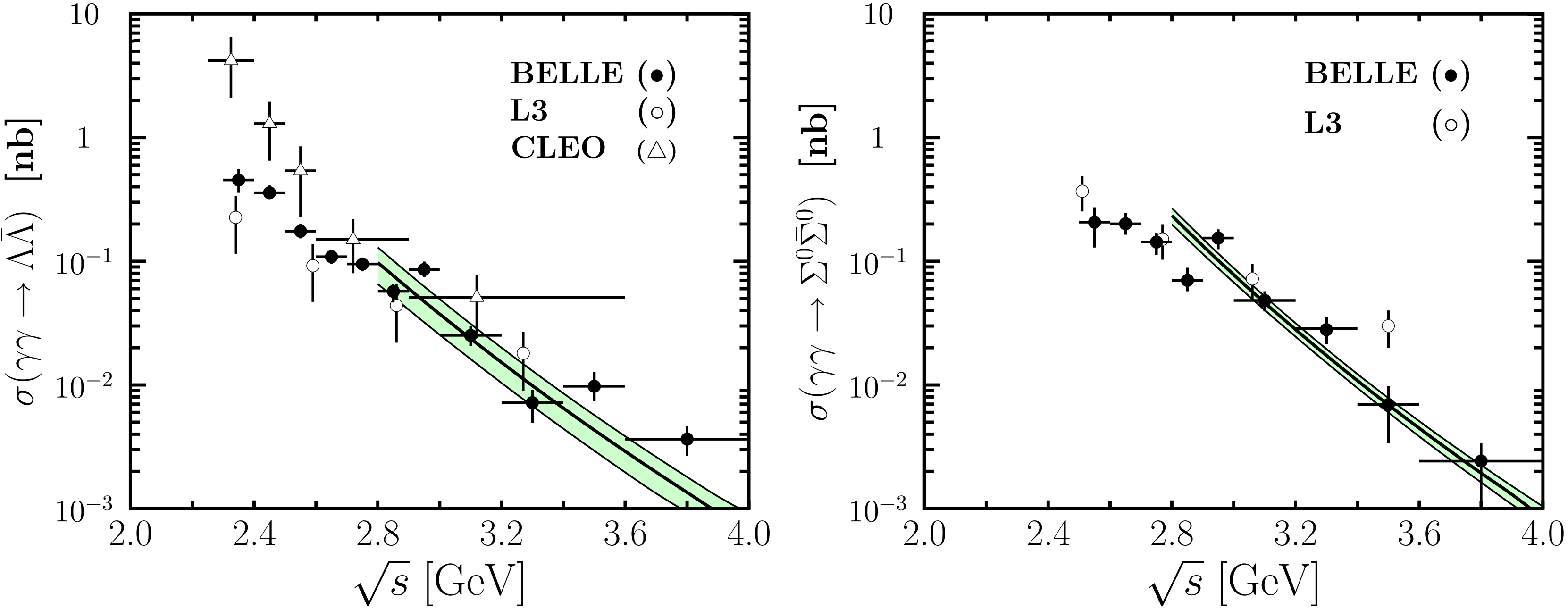
all cross sections for ground state baryons are related

(up to flavor symmetry breaking),

$$r_n = \frac{1 + 4\rho_d + \rho_s}{4 + \rho_d\rho_s} \qquad r_\Lambda = -\frac{3}{2}\frac{1 + 2\rho_d + \rho_s}{4 + \rho_d + \rho_s}$$

$$r^0_\Sigma = -\frac{1}{2}\frac{5 + 2\rho_d + 5\rho_s}{4 + \rho_d + \rho_s} \qquad r_{\Lambda\overline{\Sigma}^0} = r_{\Sigma_0\overline{\Lambda}} = \frac{\sqrt{3}}{2}\,\frac{1 - 2\rho_d + \rho_s}{4 + \rho_d + \rho_s}$$

# $\gamma\gamma \to \Lambda\overline{\Lambda}\,, \Sigma^0\overline{\Sigma}^0$




(integrated over $|\cos\theta| < 0.6$)

$\rho = 0.75 \pm 0.25 \qquad \kappa_s = -0.25\,(10.4\,\text{GeV}^2/s)$

Close-Quing duality: $\rho = 1/2$ (counting quarks)

$u(d) \to p$ fragmentation $1/4 \lesssim \rho \lesssim 1$

# Compton scattering - the form factors

in symmetric frame $\xi = \frac{p^+ - p^{+\prime}}{p^+ + p^{+\prime}} = 0$

$$R_V(t) = \sum_q e_q^2 \int_0^1 \frac{dx}{x} H_v^q(x,t)$$

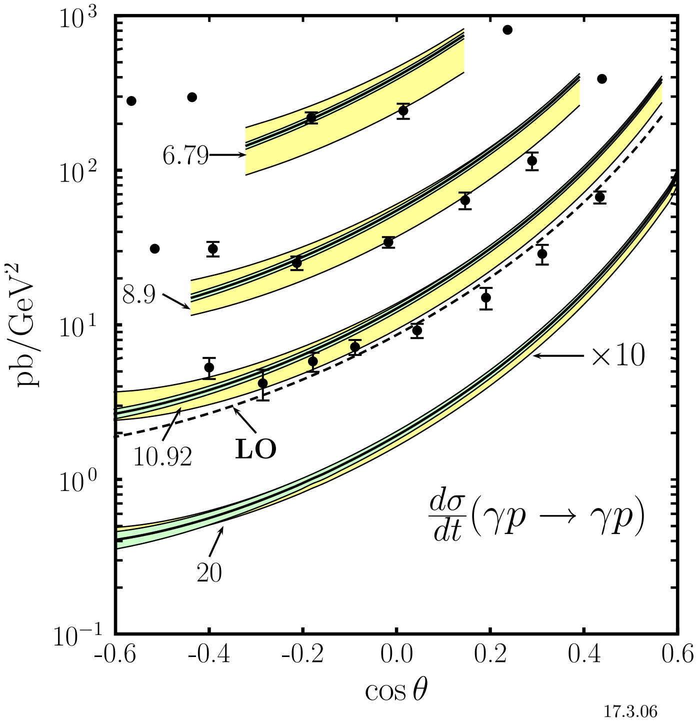
$\widetilde{H}_v^q(x,t) \Longrightarrow R_A(t)$ $E_v^q(x,t) \Longrightarrow R_T(t)$ $\widetilde{E}$ decouples

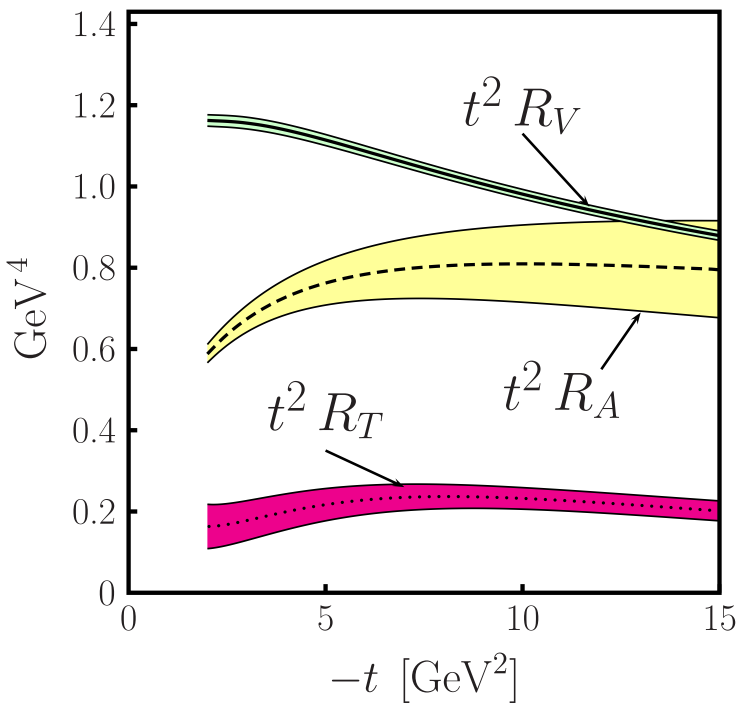
$$H_v^q = H^q - H^{\bar{q}}$$

valence quark GPDs known –
analyis of DVEM, PDFs and sum rules:

$$F_1^{p(n)}(t) \; = \int_0^1 dx \left[ e_u H_v^{u(d)}(x,t) + e_d H_v^{d(u)}(x,t) \right] \qquad F_2 \Rightarrow E, \quad F_A \Rightarrow \widetilde{H}$$

# The Compton cross section




$$\frac{d\sigma}{dt} = \frac{d\hat{\sigma}}{dt}\Big\{\frac{1}{2}\left[R_V^2(1+\kappa_T^2)+R_A^2\right]-\frac{us}{s^2+u^2}\left[R_V^2(1+\kappa_T^2)-R_A^2\right]\Big\}+\mathcal{O}(\alpha_s); \quad \kappa_T = \frac{\sqrt{-t}}{2m}\frac{R_T}{R_V}$$

$\frac{d\hat{\sigma}}{dt} = \frac{2\pi\alpha_{\rm elm}}{s^2}\left[-\frac{s}{u}-\frac{u}{s}\right]$ Klein-Nishina cross section

data: JLab E99-114

# Summary

- I presented a QCD inspired model for $\gamma\gamma \to M\bar{M}$ which is based on factorization into $\gamma\gamma \to q\bar{q}$ and 2-meson DAs
- Flavor symmetry combined with the fact that the intermediate $q\bar{q}$ state allows only $I = 0, 1$ leads to many relations among the amplitudes for the pseudoscalar channels; the soft physics is encoded in only two independent form factors
- The BELLE data on $\gamma\gamma \to M\bar{M}$ can be described very well for $s \gtrsim 9\,\mathrm{GeV}^2$.
- The approach has been extended to $B\bar{B}$ channels