Low-Energy Pion-Photon Reactions and Chiral Symmetry

N. Kaiser (TUM)

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- Tests of chiral perturbation theory via low-energy $\pi^-\gamma$ reactions
- COMPASS@CERN: Primakoff effect to extract $\pi^-\gamma$ cross sections
- Pion Compton scattering in ChPT: electric/magnetic polarizabilities
- Radiative corrections to $\pi^-\gamma \to \pi^-\gamma$
- ALICE@CERN: photon-photon fusion $\gamma\gamma \to \pi^+\pi^-$, $\pi^+\pi^-\gamma$, $\pi^+\pi^-\pi^0\pi^0$
- Neutral and charged pion-pair production: $\pi^-\gamma \to \pi^-\pi^0\pi^0$, $\pi^+\pi^-\pi^-$

Publications:

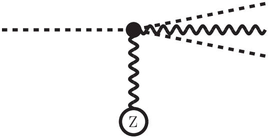
N. Kaiser, J. Friedrich, EPJ A36, 181 ('08); NPA 812, 186 ('08); EPJ A39, 71 ('09);
N. Kaiser, NPA 848, 198 ('10); EPJ A46, 373 ('10); EPJ A47, 15 ('11)

- Pions $\pi^{\pm 0}$: Goldstone bosons of spontaneous chiral symmetry breaking
- Their low-energy dynamics: systematically (and accurately) calculable in Chiral Perturbation Theory (= loop-expansion with effective Lagrangian)
- 2-loop prediction for $I = 0$ $\pi\pi$-scattering length: $a_0 m_\pi = 0.220 \pm 0.005$ confirmed by NA48/2@CERN: $K^+ \to \pi^+\pi^- e^+\nu_e$ ($\pi^+\pi^-$ mass distribut.)
- Implications: quark condensate $\langle 0|\bar{q}q|0\rangle$ is large, linear term dominates quark mass expansion of m_π^2: $m_\pi^2 f_\pi^2 = -\langle 0|\bar{q}q|0\rangle m_q + \mathcal{O}(m_q^2 \ln m_q)$
- DIRAC@CERN: Pionium lifetime $\tau_{pred} = (2.9 \pm 0.1) \cdot 10^{-15}$ sec

$$\Gamma((\pi^+\pi^-)_{atom} \to \pi^0\pi^0) = \frac{2}{9}\alpha^3 p_{cm} m_\pi^2 (a_0 - a_2)^2 + \ldots$$

- Cusp effect in $2\pi^0$ mass spectrum of $K^+ \to \pi^+\pi^0\pi^0$ at $\pi^+\pi^-$ threshold: $(a_0 - a_2)m_\pi = 0.257 \pm 0.006$, ChPT: $(a_0 - a_2)m_\pi = (0.265 \pm 0.005)$
- Electromagnetic processes with pions allow for further tests of ChPT
- Pion polarizability difference (2-loops): $\alpha_\pi - \beta_\pi = (5.7 \pm 1.0) \cdot 10^{-4}\,\text{fm}^3$, experimental determinations from Serpukhov and Mainz in conflict with it

Primakoff effect:



- Scattering of high energy pions in nuclear Coulomb field (high Z) allows to extract cross sections for $\pi^-\gamma$ reactions (equivalent-photon method)

$$\frac{d\sigma}{ds\,dQ^2} = \frac{Z^2\alpha}{\pi(s-m_\pi^2)}\,\frac{Q^2-Q^2_{min}}{Q^4}\,\sigma_{\pi^-\gamma}(s)\,, \qquad Q_{min} = \frac{s-m_\pi^2}{2E_{beam}}$$

- $s = (\pi^-\gamma \text{ invariant mass})^2$, $Q \to 0$ momentum transfer by virtual photon
- isolate Coulomb peak from strong interaction background
- COMPASS@CERN
- π-Compton scattering $\pi^-\gamma \to \pi^-\gamma$: electric and magnetic polarizabilities
- π^0-production $\pi^-\gamma \to \pi^-\pi^0$: test QCD chiral anomaly, $F_{\gamma 3\pi} = e/(4\pi^2 f_\pi^3)$
- pion-pair product. $\pi^-\gamma \to 3\pi$: $\sqrt{s} > 1$GeV meson spectroscopy, exotics, high statistics allows to continue event rates even down to threshold

Pion Compton-scattering in ChPT

- Pion Compton-scattering: $\pi^-(p_1) + \gamma(k_1, \epsilon_1) \to \pi^-(p_2) + \gamma(k_2, \epsilon_2)$
  T-matrix in center-of-mass frame in Coulomb gauge $\epsilon^0_{1,2} = 0$:

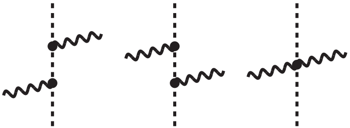
$$T_{\pi\gamma} = 8\pi\alpha\Big\{ -\vec{\epsilon}_1 \cdot \vec{\epsilon}_2\, A(s,t) + \vec{\epsilon}_1 \cdot \vec{k}_2\, \vec{\epsilon}_2 \cdot \vec{k}_1\, \frac{2}{t}\Big[A(s,t) + B(s,t)\Big]\Big\}$$

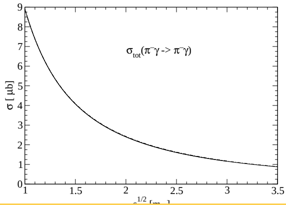
  Mandelstam variables: $s = (p_1 + k_1)^2,\ t = (k_1 - k_2)^2$
- Differential cross section:

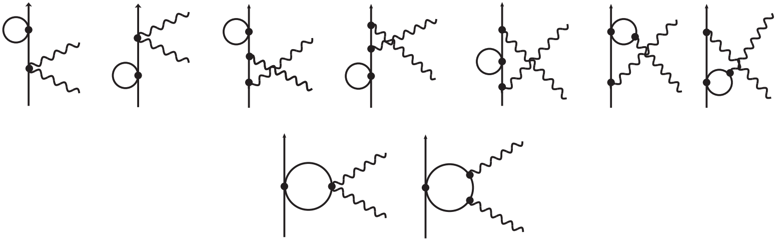
$$\frac{d\sigma}{d\Omega_{cm}} = \frac{\alpha^2}{2s}\Big\{\big|A(s,t)\big|^2 + \big|A(s,t) + (1+z)B(s,t)\big|^2\Big\}$$

  $t = (s - m_\pi^2)^2(z-1)/2s$ with $z = \cos\theta_{cm}$, scattering angle
- Tree diagrams:

$$A(s,t)^{(tree)} = 1, \qquad B(s,t)^{(tree)} = \frac{s - m_\pi^2}{m_\pi^2 - s - t}$$







- <u>Pion-loop diagrams</u> (photon scattering off the pion's "pion cloud"):

$$A(s,t)^{(loop)} = \frac{1}{(4\pi f_\pi)^2}\left\{-\frac{t}{2} - 2m_\pi^2 \ln^2 \frac{\sqrt{4m_\pi^2 - t} + \sqrt{-t}}{2m_\pi}\right\} \sim t^2 > 0$$

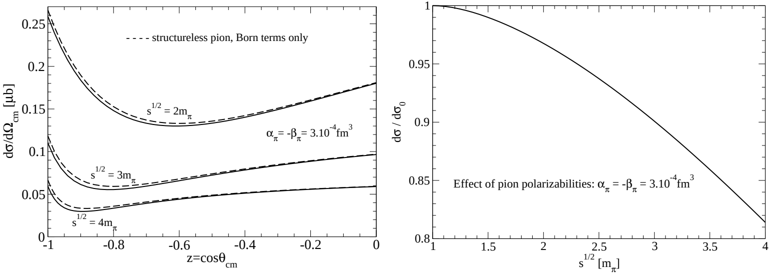
with $f_\pi = 92.4\,\text{MeV}$, expression corresponds to isospin limit: $m_{\pi^0} = m_\pi$

- <u>Electric/magnetic polarizabilities</u> = low-energy const. with $\alpha_\pi + \beta_\pi = 0$

$$A(s,t)^{(pola)} = -\frac{\beta_\pi m_\pi t}{2\alpha} < 0\,, \qquad \alpha_\pi - \beta_\pi = \frac{\alpha}{24\pi^2 f_\pi^2 m_\pi}(\bar\ell_6 - \bar\ell_5)$$

Pion Compton-scattering in ChPT

- Combination $\bar{\ell}_6 - \bar{\ell}_5 = 3.0 \pm 0.3$ determined via radiative pion decay $\pi^+ \to e^+\nu_e\gamma$, PIBETA@PSI: axial-to-vector coupl. ratio $F_A/F_V \simeq 0.44$
- Current-algebra relation: $\langle 0|A^\mu V^\nu|\pi\rangle \simeq f_\pi \langle \pi|V^\mu V^\nu|\pi\rangle$ plus corrections
- One-loop "prediction": $\alpha_\pi = -\beta_\pi \simeq 3.0 \cdot 10^{-4}\,\mathrm{fm}^3$
- $\sigma_{tot}(s)$ insensitive to pion's low-energy structure
- Small effect on backward angular distributions of $d\sigma/d\Omega_{cm}$



- Pion-loop compensates partly reduction of $d\sigma/d\Omega_{cm}$ by polarizabilities
- Effect of pion polarizabilities on π-Compton cross section: less than 20%
- 2-loop corrections to $d\sigma/d\Omega_{cm}$ are very small (Gasser, Ivanov)

Pion polarizabilities in ChPT

- Gasser et al., NPB745, 84 (2006): Pion polarizabilities to 2 loops
- Analytical expression in terms of low-energy constants $\bar{\ell}_j$:

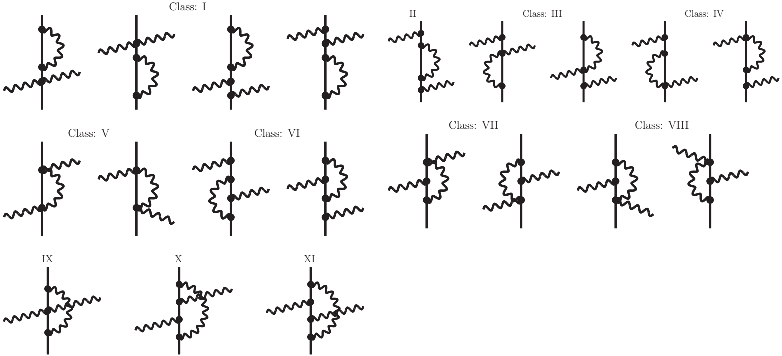
$$\alpha_\pi - \beta_\pi = \frac{\alpha(\bar{\ell}_6 - \bar{\ell}_5)}{24\pi^2 f_\pi^2 m_\pi} + \frac{\alpha m_\pi}{(4\pi f_\pi)^4}\Big\{c^r + \frac{8}{3}\Big(\bar{\ell}_2 - \bar{\ell}_1 + \bar{\ell}_5 - \bar{\ell}_6 + \frac{65}{12}\Big)\ln\frac{m_\pi}{m_\rho}$$
$$+\frac{4}{9}(\bar{\ell}_1 + \bar{\ell}_2) - \frac{\bar{\ell}_3}{3} + \frac{4\bar{\ell}_4}{3}(\bar{\ell}_6 - \bar{\ell}_5) - \frac{187}{81} + \Big(\frac{53\pi^2}{48} - \frac{41}{324}\Big)\Big\}$$

- Improved values of $\bar{\ell}_j$ from $\pi\pi$ data, $c^r \simeq 0$ via resonance saturation
- 2-loop prediction including realistic estimate of theoretical errors:

$$\alpha_\pi - \beta_\pi = (5.7 \pm 1.0)\cdot 10^{-4}\,\text{fm}^3\,, \qquad \alpha_\pi + \beta_\pi = (0.16 \pm 0.1)\cdot 10^{-4}\,\text{fm}^3$$

- Good reasons to believe that chiral prediction is stable against higher order corrections: ChPT at 2-loop order works very well for $\gamma\gamma \to \pi^0\pi^0$
- Existing expt. determinations $\alpha_\pi - \beta_\pi = (15.6 \pm 7.8)\cdot 10^{-4}\,\text{fm}^3$ from Serpukhov (via Primakoff) and $\alpha_\pi - \beta_\pi = (11.6 \pm 3.4)\cdot 10^{-4}\,\text{fm}^3$ from Mainz (via $\gamma p \to \gamma\pi^+ n$) violate chiral low-energy theorem by a factor 2!
- $\alpha_\pi + \beta_\pi = \frac{1}{2\pi^2}\int \frac{d\omega}{\omega^2}\sigma_{\text{abs}}^{\pi\gamma}(\omega)$ agrees with results from dispersion sum rules

- Pion-structure effects small: necessary to include radiative corr. of $\mathcal{O}(\alpha)$
- Start with structureless pion: extensive calculation in 1-loop scalar QED
- Advantage of Coulomb gauge: all s-channel pole diagrams vanish

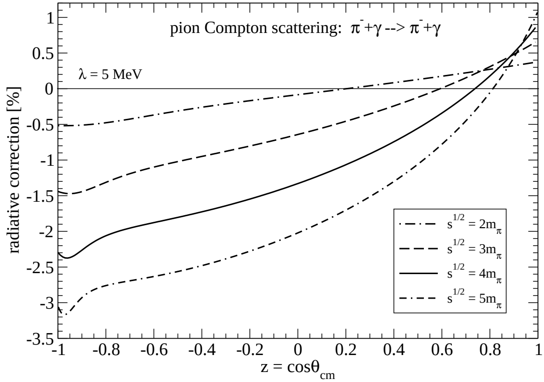


- Infrared-finite after inclusion of soft photon bremsstrahlung: $d\sigma/d\Omega \cdot \delta_{\rm soft}$

$$\delta_{\rm soft} = \alpha\, \mu^{4-d} \int\limits_{|\vec{l}\,|<\lambda} \frac{d^{d-1}l}{(2\pi)^{d-2}\, l_0} \left\{ \frac{2m_\pi^2 - t}{p_1\cdot l\, p_2\cdot l} - \frac{m_\pi^2}{(p_1\cdot l)^2} - \frac{m_\pi^2}{(p_2\cdot l)^2} \right\}$$

Radiative corrections to pion Compton scattering

Results:



- QED radiative corrections are maximal in backward directions $z \simeq -1$
- Same kinematical signature as pion polarizability difference $\alpha_\pi - \beta_\pi$
- Suppressed by a factor of $\lesssim 10$
- In long wavelength limit $k_1, k_2 \to 0$: all strong and radiative corrections vanish, pure Thomson amplitude $T^{(0)}_{\pi^-\gamma} = -8\pi\alpha\,\vec{\epsilon}_1 \cdot \vec{\epsilon}_2$ survives

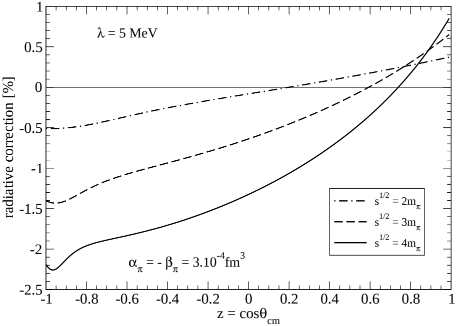
Radiative corrections including pion structure

- Include leading pion-structure in form of polarizability difference $\alpha_\pi - \beta_\pi$
- Reinterpret $\gamma\gamma$ contact vertex as representing the pion polarizabilities:

$$\sim F_{\mu\nu}F^{\mu\nu}\,, \qquad 8\pi i\,\beta_\pi m_\pi \Big(k_1\cdot k_2\,\epsilon_1\cdot\epsilon_2 - \epsilon_1\cdot k_2\,\epsilon_2\cdot k_1\Big)$$

- Reference cross section: point-like $d\sigma^{(pt)}/d\Omega_{cm}$ + polarizability improved

$$\frac{d\sigma^{(\mathrm{pola})}}{d\Omega_{\mathrm{cm}}} = \frac{\alpha\beta_\pi m_\pi^3(s-m_\pi^2)^2(1-z)^2}{2s^2[s(1+z)+m_\pi^2(1-z)]}$$



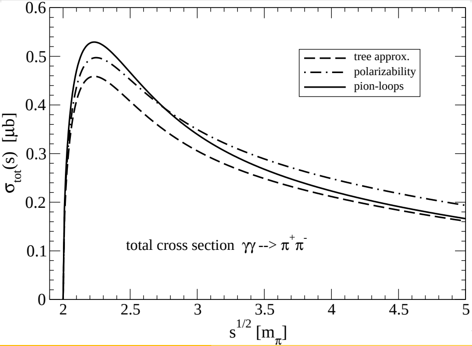
- Relative size and angular depend. <u>not affected</u> by leading pion-structure

Photon-photon fusion process $\gamma\gamma \to \pi^+\pi^-$

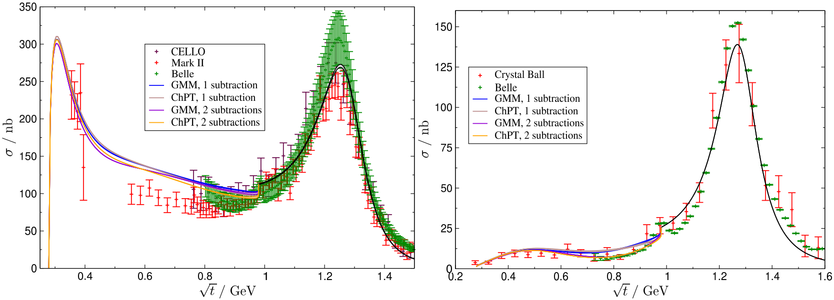
- ALICE@CERN: Ultraperipheral nucleus-nucleus collisions, $Pb^{82+}Pb^{82+}$
- $\gamma\gamma \to \pi^+\pi^-$ is crossed reaction to pion Compton scattering, $\hat{s} = s/m_\pi^2$

$$\sigma_{tot}(s) = \frac{2\pi\alpha^2}{\hat{s}^3 m_\pi^2}\Big\{\left[4+\hat{s}+\hat{s}|C(\hat{s})|^2\right]\sqrt{\hat{s}(\hat{s}-4)}$$
$$+8\left[2-\hat{s}+\hat{s}\,\mathrm{Re}C(\hat{s})\right]\ln\frac{\sqrt{\hat{s}}+\sqrt{\hat{s}-4}}{2}\Big\},$$

$$C(\hat{s}) = -\beta_\pi \frac{m_\pi^3}{2\alpha}\hat{s} - \frac{m_\pi^2}{(4\pi f_\pi)^2}\left\{\frac{\hat{s}}{2} + 2\left[\ln\frac{\sqrt{\hat{s}}+\sqrt{\hat{s}-4}}{2} - \frac{i\pi}{2}\right]^2\right\}$$



- Roy-Steiner equations (dispersion relations) for pion Compton scattering respect fully analyticity, unitarity, crossing symmetry & gauge invariance
- Complete treatment of $\pi\pi$ final state interaction via $\pi\pi$ phase shift solut., subtracted dispersion relations reduce sensitivity to high-energy input, $\to$ pion polarizabilities α_π, β_π etc. serve as subtraction constants



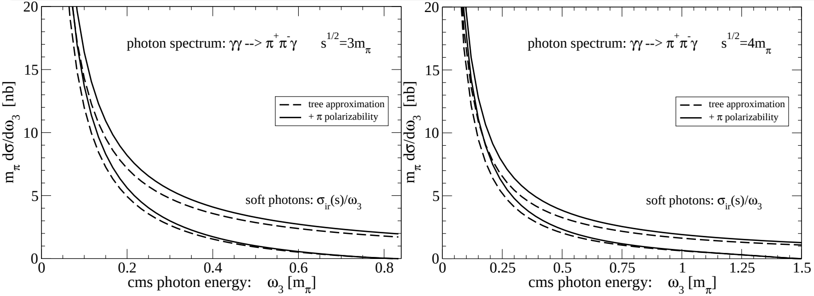
- Hoferichter et al., EPJ C71, 1743 (2011); Garcia-Martin et al., EPJ C70, 155 ('10)
- expt. cross section: $\gamma\gamma \to \pi^+\pi^-, |\cos\theta| < 0.6$, $\gamma\gamma \to \pi^0\pi^0, |\cos\theta| < 0.8$
- Need for improved $\gamma\gamma \to 2\pi$ data in region $\sqrt{s} < 1$GeV below $f_2(1270)$

- Photon spectrum of the process $\gamma\gamma \to \pi^+\pi^-\gamma_3$, cms photon energy ω_3

$$\frac{d\sigma}{d\omega_3} = \frac{\alpha^3}{2\pi s}\int_{\omega_1^-}^{\omega_1^+} d\omega_1 \int_{-1}^{1} dx \int_0^{\pi} d\phi\,(-T^{\mu\nu\rho}T_{\mu\nu\rho})\,, \quad T_{\mu\nu\rho} \textit{ transversal}$$

- Soft photon approximation: $\omega_3 \ll (s - 4m_\pi^2)/2\sqrt{s}$

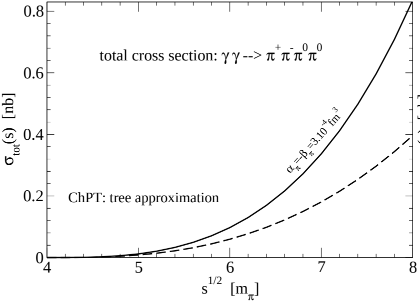
$$\frac{d\sigma}{d\omega_3} = \frac{1}{\omega_3}\frac{2\alpha}{\pi}\left\{\frac{2\hat{s}-4}{\sqrt{\hat{s}(\hat{s}-4)}}\ln\frac{\sqrt{\hat{s}}+\sqrt{\hat{s}-4}}{2} - 1\right\}\sigma_{tot}^{\gamma\gamma\to\pi^+\pi^-}(s) + \dots$$

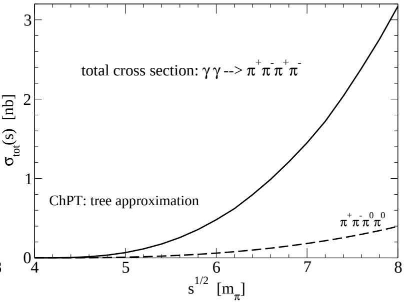


- 10 -15 % enhancement by pion polarizability difference $\alpha_\pi - \beta_\pi$

- At leading order governed by chiral $\pi\pi$ interaction (and $\pi^\pm\gamma$-coupling)
- Chiral $\pi^+\pi^- \to \pi^0\pi^0$ amplitude depends only on $2\pi^0$ invariant mass μ_0

$$\sigma_{tot}(s) = \frac{\alpha^2}{8s\,f_\pi^4}\int_{2m_\pi}^{\sqrt{s}-2m_\pi} d\mu_0 \sqrt{\mu_0^2 - 4m_\pi^2}\,(\mu_0^2 - m_\pi^2)^2 \int d\Phi_3\, t^{\mu\nu} t_{\mu\nu}$$

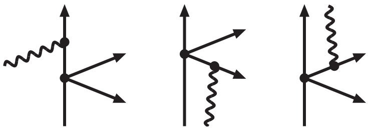


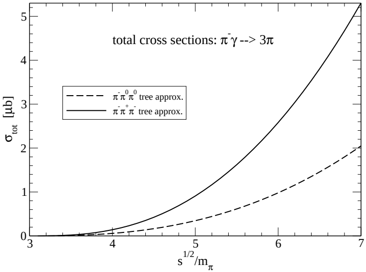


- Effect of pion polarizability difference $\alpha_\pi - \beta_\pi$ sizeable (extrapolation?)
- Tree approximation should be considered as first rough estimate (loops?)
- $\rho(770)$ resonance should be included consistently with chiral symmetry

Tree level cross sections for $\pi^-\gamma \to 3\pi$

- Coulomb gauge $\epsilon \cdot p_1 = \epsilon \cdot k = 0$, photon does not couple to incoming π^-
- No $\gamma 4\pi$ vertex at leading order





- Example: total cross section for $\pi^-(p_1) + \gamma(k,\epsilon) \to \pi^-\pi^0\pi^0$

$$\sigma_{tot}(s) = \frac{\alpha}{32\pi^2 f_\pi^4 (s-m_\pi^2)^3} \int_{2m_\pi\sqrt{s}}^{s-3m_\pi^2} dw \sqrt{\frac{s-w-3m_\pi^2}{s-w+m_\pi^2}}\,(s-w)^2$$
$$\times \left[w \ln \frac{w+\sqrt{w^2-4m_\pi^2 s}}{2m_\pi\sqrt{s}} - \sqrt{w^2-4m_\pi^2 s} \right]$$

- $(s-w)/f_\pi^2$ factor: chiral $\pi\pi$-interaction, rest from 3-body phase space
- How large are next-to-leading order corrections from chiral loops + cts?

Neutral pion-pair production

- 3-body process: $\pi^-(p_1) + \gamma(k,\epsilon) \to \pi^-(p_2) + \pi^0(q_1) + \pi^0(q_2)$
- general form of T-matrix (in Coulomb gauge)

$$T_{3\pi} = \frac{2e}{f_\pi^2}\left[\vec{\epsilon}\cdot\vec{q}_1 A_1 + \vec{\epsilon}\cdot\vec{q}_2 A_2\right], \qquad A_2 = A_1\Big|(s_1 \leftrightarrow s_2, t_1 \leftrightarrow t_2)$$

- amplitudes A_1 and A_2 depend on five (independ.) Mandelstam variables:

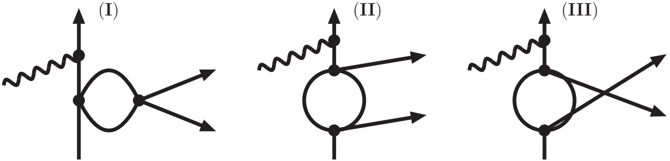
$$s = (p_1+k)^2,\ \ s_1 = (p_2+q_1)^2,\ \ s_2 = (p_2+q_2)^2,\ \ t_1 = (q_1-k)^2,\ \ t_2 = (q_2-k)^2$$

- convenient for permutation of identical neutral pions ($s_1 \leftrightarrow s_2$, $t_1 \leftrightarrow t_2$)
- tree-level amplitudes:

$$A_1^{(\mathrm{tree})} = A_2^{(\mathrm{tree})} = \frac{2m_\pi^2 + s - s_1 - s_2}{3m_\pi^2 - s - t_1 - t_2}$$

Neutral pion-pair production

- Pion-loop corrections (example I)



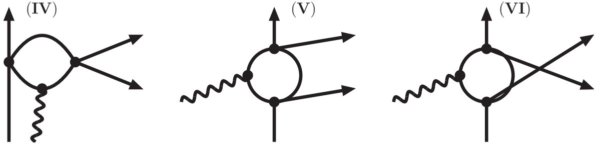
$$A_1^{(\mathrm{I})} = \frac{1}{(4\pi f_\pi)^2}\frac{2m_\pi^2+s-s_1-s_2}{3m_\pi^2-s-t_1-t_2}\left\{\left(\xi+\ln\frac{m_\pi}{\mu}\right)(s_1+s_2+t_1+t_2-11m_\pi^2)\right.$$
$$\left.+(s_1+s_2+t_1+t_2-7m_\pi^2)\left[J(3m_\pi^2+s-s_1-s_2)-\frac{1}{2}\right]\right\}$$

- Loop function (from loop with two pion-propagators)

$$J(s)=\sqrt{\frac{s-4m_\pi^2}{s}}\left[\ln\frac{\sqrt{|s-4m_\pi^2|}+\sqrt{|s|}}{2m_\pi}-\frac{i\pi}{2}\theta(s-4m_\pi^2)\right],\quad s<0 \text{ or } s>4m_\pi^2$$

Neutral pion-pair production

- Pion-loop corrections (example IV)



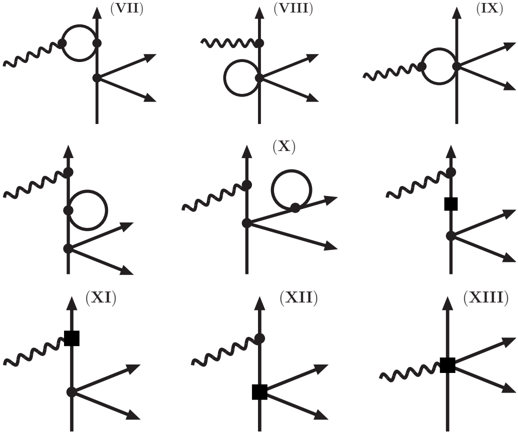
$$A_1^{(\mathrm{IV})} = \frac{2m_\pi^2+s-s_1-s_2}{(4\pi f_\pi)^2}\Bigg\{\xi+\ln\frac{m_\pi}{\mu}-\frac{1}{2}+J(3m_\pi^2+s-s_1-s_2)$$
$$+\frac{m_\pi^2-s}{2m_\pi^2-t_1-t_2}+\frac{2(s-m_\pi^2)}{(2m_\pi^2-t_1-t_2)^2}\Big\{(s_1+s_2-s-m_\pi^2-t_1-t_2)$$
$$\times\Big[J(m_\pi^2+s-s_1-s_2+t_1+t_2)-J(3m_\pi^2+s-s_1-s_2)\Big]$$
$$+2m_\pi^2\Big[G(m_\pi^2+s-s_1-s_2+t_1+t_2)-G(3m_\pi^2+s-s_1-s_2)\Big]\Big\}\Bigg\}$$

- Loop function (from loop with three pion-propagators)

$$G(s)=\left[\ln\frac{\sqrt{|s-4m_\pi^2|}+\sqrt{|s|}}{2m_\pi}-\frac{i\pi}{2}\theta(s-4m_\pi^2)\right]^2, \quad s<0 \text{ or } s>4m_\pi^2$$

Neutral pion-pair production

- Chiral loop and counterterm corrections (completed)



- Chiral 6π-vertex: challenging combinatorics involved, $6! = 720$
- Pion wavefunction renormalization factor, chiral counterterms $\sim \ell_1, \ell_2, \ell_4$
- First crucial check: ultraviolet divergence ξ drops out in total sum for $A_{1,2}$

Neutral pion-pair production

- Introduce low-energy constants that subsume chiral logarithm $\ln(m_\pi/\mu)$

$$\ell_j^r = \frac{\gamma_j}{32\pi^2}\left(\bar{\ell}_j + 2\ln\frac{m_\pi}{\mu}\right), \quad \gamma_1 = \frac{1}{3},\ \gamma_2 = \frac{2}{3},\ \gamma_3 = -\frac{1}{2},\ \gamma_4 = 2$$

- Complete counterterm contribution:

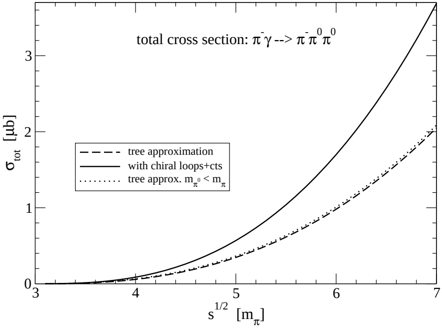
$$A_1^{(\mathrm{ct})} = \frac{1}{(4\pi f_\pi)^2}\frac{1}{3m_\pi^2 - s - t_1 - t_2}\left\{\frac{\bar{\ell}_1}{3}(s_1 + s_2 - s - m_\pi^2)^2 + \frac{\bar{\ell}_2}{3}\Big[s^2 + s_1^2 + s_2^2 + t_2^2 - 2ss_1 + (s - 2s_1 + 2s_2 - t_1)t_2 + m_\pi^2(s - 6s_2 + t_1 - 2t_2 + 6m_\pi^2)\Big] - \frac{\bar{\ell}_3}{2}m_\pi^4 + 2\bar{\ell}_4 m_\pi^2(s + 2m_\pi^2 - s_1 - s_2)\right\}$$

- Finite loop corrections with $\xi + \ln(m_\pi/\mu)$ terms deleted altogether
- Values of low-energy constants: $\bar{\ell}_1 = -0.4 \pm 0.6$, $\bar{\ell}_2 = 4.3 \pm 0.1$, $\bar{\ell}_3 = 2.9 \pm 2.4$, $\bar{\ell}_4 = 4.4 \pm 0.2$, determined with improved empirical input
- Uncertainty induced by errorbars of $\bar{\ell}_j$: about $\pm 5\%$ for $\sigma_{\mathrm{tot}}(s)$, mainly $\bar{\ell}_1$

Neutral pion-pair production

- Total cross section for $\pi^-\gamma \to 3\pi$

$$\sigma_{\rm tot}(s) = \frac{\alpha}{32\pi^3 f_\pi^4 (s-m_\pi^2)} \iint\limits_{z^2<1} d\omega_1 d\omega_2 \int_{-1}^{1} dx \int_0^\pi d\phi \left|\hat{k}\times(\vec{q}_1 A_1 + \vec{q}_2 A_2)\right|^2$$

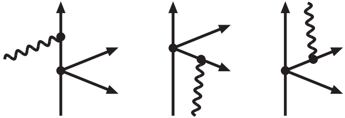


- enhancement of $\sigma_{\rm tot}(s)$ by factor 1.5 - 1.8 through chiral corrections
- suggestive explanation: $\pi^+\pi^- \to \pi^0\pi^0$ final state interaction $(1+0.20)^2$

$$\frac{1}{3}(a_0 - a_2) = \frac{3m_\pi}{32\pi f_\pi^2}\left[1 + \frac{m_\pi^2}{36\pi^2 f_\pi^2}\left(\bar{\ell}_1 + 2\bar{\ell}_2 - \frac{3\bar{\ell}_3}{8} + \frac{9\bar{\ell}_4}{2} + \frac{33}{8}\right)\right]$$

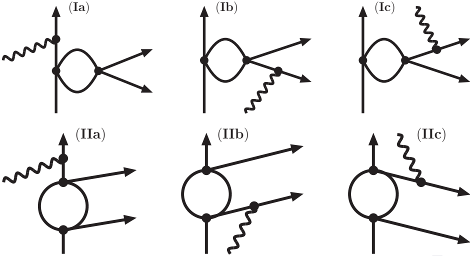
Charged pion-pair production

- 3-body process: $\pi^-(p_1) + \gamma(k, \epsilon) \to \pi^+(p_2) + \pi^-(q_1) + \pi^-(q_2)$
- Photon couples to all charged pions: $\to$ many more diagrams



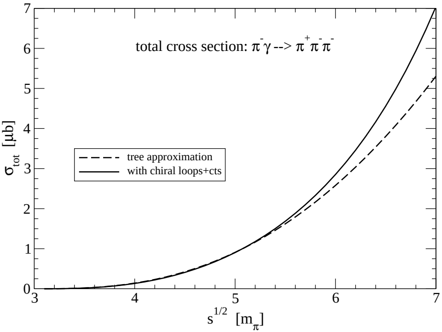
$$A_1^{(\text{tree})} = \frac{s + m_\pi^2 - s_1 - s_2}{3m_\pi^2 - s - t_1 - t_2} + \frac{s - s_1 - s_2 + t_2}{t_1 - m_\pi^2} - 1$$

$$A_2^{(\text{tree})} = \frac{s + m_\pi^2 - s_1 - s_2}{3m_\pi^2 - s - t_1 - t_2} + \frac{s - s_1 - s_2 + t_1}{t_2 - m_\pi^2} - 1$$



Charged pion-pair production

- Total cross section

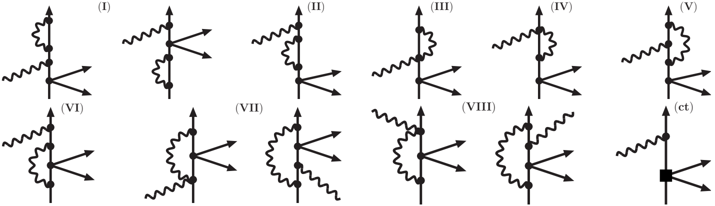


- $\sigma_{\rm tot}(s)$ for $\sqrt{s} < 6m_\pi$ almost unchanged in comparison to tree approx.
- suggestive explanation: $\pi^-\pi^- \to \pi^-\pi^-$ final state interaction $(1-0.02)^2$

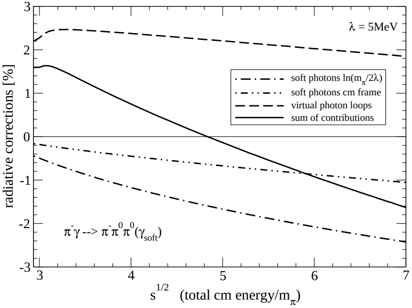
$$a_2 = -\frac{m_\pi}{16\pi f_\pi^2}\left[1 - \frac{m_\pi^2}{12\pi^2 f_\pi^2}\left(\bar{\ell}_1 + 2\bar{\ell}_2 - \frac{3\bar{\ell}_3}{8} - \frac{3\bar{\ell}_4}{2} + \frac{3}{8}\right)\right]$$

- Analysis of COMPASS data for $\sqrt{s} \leq 5m_\pi$ agrees with ChPT prediction
  First measurement of chiral dynamics in $\pi^-\gamma \to \pi^-\pi^-\pi^+$, hep-ex/1111.5954

- Chiral $\pi^+\pi^- \to \pi^0\pi^0$ transition amplitude factors out of all photon loops



- Radiative corr. to total cross section vary between about $+2\%$ and -2%



- Radiative corrections to $\pi^-\gamma \to \pi^+\pi^-\pi^-$ could be more sizeable, soft photon part is <u>similar</u>, but Coulomb singularity from γ-exchange: $\alpha\pi/v_{\rm rel}$