Mueller-Navelet jet production in proton-proton collisions

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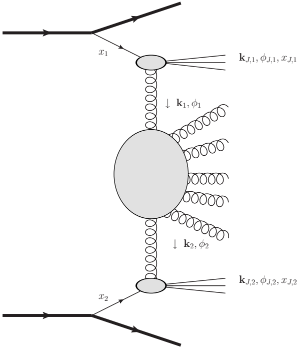
Dijet production process

$$p(p_1) + p(p_2) \to J_1(k_{J,1}) + J_2(k_{J,2}) + X$$

considered in special kinematics



where the jets are separated by a large interval of rapidity, $\Delta y \gg 1$.
This regime requires large center of mass energy s of the proton collisions, $s = 2p_1 \cdot p_2 \gg \vec{k}^2_{J\,1,2}$, since $\Delta y \sim \ln s/\vec{k}^2_{J\,1,2}$.

QCD description of hard processes, hard scale $Q^2 \gg \Lambda^2_{QCD}$,
in our case $Q^2 \sim \vec{k}^2_{J\,1,2}$.
Neglecting power suppressed contributions $\sim 1/Q$, we introduce leading twist PDFs: $f_g(x)$ and $f_q(x)$.

Still need to resum QCD perturbation series

- DGLAP:
  $\sim \sum_n a_n \alpha_s^n \ln^n Q^2 + b_n \alpha_s^n \ln^{n-1} Q^2$
- BFKL:
  $\sim \sum_n c_n \alpha_s^n \ln^n s + d_n \alpha_s^n \ln^{n-1} s$

Mueller-Navelet idea: At $\Delta y \gg 1$ BFKL approach can be more adequate.

- "Faster" energy dependence
- More decorrelation in relative jets azimuthal angle $\phi = \phi_{J,1} - \phi_{J,2} - \pi$ is expected.

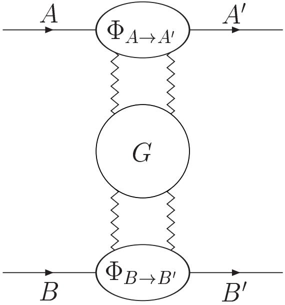
Contents

- BFKL method
- NLA jet vetex and CSSW results
- NLA jet vertex in small cone approximation
- Similar processes. NLA vertex for identified hadron production.
- Summary and prospects

BFKL method

Scattering $A + B \longrightarrow A' + B'$ in the Regge kinematical region $s \to \infty$, t fixed

BFKL approach: convolution of the Green's function of two interacting Reggeized gluons and of the impact factors of the colliding particles.



Valid both in
LLA (resummation of all terms $(\alpha_s \ln(s))^n$)
NLA (resummation of all terms $\alpha_s(\alpha_s \ln(s))^n$).

The Green's function is determined through the BFKL equation.
[Ya.Ya. Balitsky, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

The kernel of the BFKL equation is completely known in the NLA for the forward $(t = 0)$ case.
[V.S. Fadin, L.N. Lipatov (1998)]
[G. Camici, M. Ciafaloni (1998)]

Impact factors

have been calculated in the NLA for

- colliding partons [V.S. Fadin, R. Fiore, M.I. Kotsky, A.P. (2000)] [M. Ciafaloni and G. Rodrigo (2000)]

Colorless NLA impact factors

- forward jet production [J. Bartels, D. Colferai, G.P. Vacca (2003)] [F. Caporale, D. I., B. Murdaca, A. Papa, A. Perri. (2011)]
- $\gamma^* \to \gamma^*$, in impact parameter space [J. Balitsky, Chirili (2011)]
- $\gamma^* \to V$, with $V = \rho^0$, ω, ϕ, forward case [D.Yu. Ivanov, M.I. Kotsky, A. P. (2004)]

$$\mathcal{I}m_s\left(\mathcal{A}\right) = \frac{s}{(2\pi)^2}\int\frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}}\Phi_1(\vec{q}_1, s_0)\int\frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}}\Phi_2(-\vec{q}_2, s_0)\int\limits_{\delta-i\infty}^{\delta+i\infty}\frac{d\omega}{2\pi i}\left(\frac{s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$

BFKL equation: $\delta^2(\vec{q}_1 - \vec{q}_2) = \omega\, G_\omega(\vec{q}_1, \vec{q}_2) - \int d^2\vec{q}\, K(\vec{q}_1, \vec{q})\, G_\omega(\vec{q}, \vec{q}_2)$

Transverse momentum notation: $\hat{\vec{q}}\,|\vec{q}_i\rangle = \vec{q}_i|\vec{q}_i\rangle$

$$\langle\vec{q}_1|\vec{q}_2\rangle = \delta^{(2)}(\vec{q}_1 - \vec{q}_2) \qquad \langle A|B\rangle = \langle A|\vec{k}\rangle\langle\vec{k}|B\rangle = \int d^2k A(\vec{k})B(\vec{k})$$

$$\hat{1} = (\omega - \hat{K})\hat{G}_\omega \qquad \longrightarrow \qquad \hat{G}_\omega = (\omega - \hat{K})^{-1}$$

$$\hat{K} = \bar{\alpha}_s\hat{K}^0 + \bar{\alpha}_s^2\hat{K}^1\,, \qquad \bar{\alpha}_s = \frac{\alpha_s N_c}{\pi}$$

With NLA accuracy

$$\hat{G}_\omega = (\omega - \bar{\alpha}_s\hat{K}^0)^{-1} + (\omega - \bar{\alpha}_s\hat{K}^0)^{-1}\left(\bar{\alpha}_s^2\hat{K}^1\right)(\omega - \bar{\alpha}_s\hat{K}^0)^{-1} + \mathcal{O}\left[\left(\bar{\alpha}_s^2\hat{K}^1\right)^2\right]$$

Basis of eigenfunctions of the LLA kernel: $\{|n,\nu\rangle\}$

$$\boxed{\hat{K}^0|n,\nu\rangle = \chi(n,\nu)|n,\nu\rangle}$$

$$\chi(n,\nu) = 2\psi(1) - \psi\left(\tfrac{n}{2}+\tfrac{1}{2}+i\nu\right) - \psi\left(\tfrac{n}{2}+\tfrac{1}{2}-i\nu\right)$$

$$\langle\vec{q}|n,\nu\rangle = \frac{1}{\pi\sqrt{2}}\left(\vec{q}^{\,2}\right)^{i\nu-\frac{1}{2}} e^{in\phi}$$

$$\langle\nu',n'|n,\nu\rangle = \int \frac{d^2\vec{q}}{2\pi^2}\left(\vec{q}^{\,2}\right)^{i\nu-i\nu'-1} e^{i(n-n')\phi} = \delta(\nu-\nu')\delta_{n,n'}$$

Action of the full NLA kernel on the LLA eigenfunctions:

$$\begin{aligned}
\hat{K}|n,\nu\rangle &= \bar{\alpha}_s(\mu_R)\chi(n,\nu)|n,\nu\rangle \\
&+ \bar{\alpha}_s^2(\mu_R)\left(\chi^{(1)}(n,\nu) + \frac{\beta_0}{4N_c}\chi(n,\nu)\ln(\mu_R^2)\right)|n,\nu\rangle \\
&+ \bar{\alpha}_s^2(\mu_R)\frac{\beta_0}{4N_c}\chi(n,\nu)\left(i\frac{\partial}{\partial\nu}\right)|n,\nu\rangle
\end{aligned}$$

$\chi_1(n,\nu)$ is known [Kotikov, Lipatov].

$$
d\sigma(s,\phi) = \frac{1}{(2\pi)^2}\sum_n \int\limits_{-\infty}^{+\infty} d\nu \left(\frac{s}{s_0}\right)^{\bar\alpha_s(\mu_R)\chi(n,\nu)} \alpha_s^2(\mu_R) c_1(n,\nu) c_2(n,\nu)
$$

$$
\times \left[1 + \bar\alpha_s(\mu_R)\left(\frac{c_1^{(1)}(n,\nu)}{c_1(n,\nu)} + \frac{c_2^{(1)}(n,\nu)}{c_2(n,\nu)}\right) + \bar\alpha_s^2(\mu_R)\ln\left(\frac{s}{s_0}\right)\left(\bar\chi(n,\nu)\right.\right.
$$

$$
\left.\left. + \frac{\beta_0}{8N_c}\chi(n,\nu)\left[-\chi(n,\nu) + \frac{10}{3} + i\frac{d\ln\left(\frac{c_1(n,\nu)}{c_2(n,\nu)}\right)}{d\nu} + 2\ln(\mu_R^2)\right]\right)\right]
$$

$|n,\nu\rangle$ representation for impact factors:

$$
c_1(n,\nu) = \int d^2\vec q\, \Phi_1^{(0)}(\vec q^{\,2}) \frac{\left(\vec q^{\,2}\right)^{i\nu-\frac{3}{2}}}{\pi\sqrt{2}} e^{in\phi};\ c_2(n,\nu) = \int d^2\vec q\, \Phi_2^{(0)}(\vec q^{\,2}) \frac{\left(\vec q^{\,2}\right)^{-i\nu-\frac{3}{2}}}{\pi\sqrt{2}} e^{-in\phi}
$$

(analogous definitions for $c_1^{(1)}(\nu)$ and $c_2^{(1)}(\nu)$)

NLA jet vertex

[J. Bartels, D. Colferai, G.P. Vacca (2003)]
[F. Caporale, D. I., B. Murdaca, A. Papa, A. Perri. (2011)]

$$
\begin{aligned}
&V_{\rm g}^{(1)}(\mathbf{k},x) \\
= \quad & \left[\left(\frac{11}{6}\frac{C_A}{\pi}-\frac{1}{3}\frac{N_f}{\pi}\right)\ln\frac{\mathbf{k}^2}{\Lambda^2}+\left(\frac{\pi^2}{4}-\frac{67}{36}\right)\frac{C_A}{\pi}+\frac{13}{36}\frac{N_f}{\pi}-b_0\ln\frac{\mathbf{k}^2}{\mu^2}\right]V_{\rm g}^{(0)}(\mathbf{k},x) \\
&+\int {\rm d}z\,\frac{N_f}{\pi}\frac{C_F}{C_A}z(1-z)V_{\rm g}^{(0)}(\mathbf{k},xz) \\
&+\frac{N_f}{\pi}\int\frac{{\rm d}^2\mathbf{k}'}{\pi}\int_0^1 {\rm d}z\,P_{\rm qg}(z)\left[\frac{h_{\rm q}^{(0)}(\mathbf{k}')}{(\mathbf{k}-\mathbf{k}')^2+\mathbf{k}'^2}\mathcal{S}_J^{(3)}(\mathbf{k}',\mathbf{k}-\mathbf{k}',xz;x)\right. \\
&\qquad\qquad\left.-\frac{1}{\mathbf{k}'^2}\Theta(\Lambda^2-\mathbf{k}'^2)V_{\rm q}^{(0)}(\mathbf{k},xz)\right]+\dots
\end{aligned}
\tag{1}
$$

- complicated... Needs to be projected on $|n,\nu\rangle$
- requires involved numerical integration ...

NLA results of [Colferai, Szymanowski, Schwensen, Wallon (2010)]

cross section integrated over ϕ, $\frac{d^2\sigma}{dk_{j,1}dk_{J,2}}[\frac{\text{nb}}{\text{GeV}^2}]$

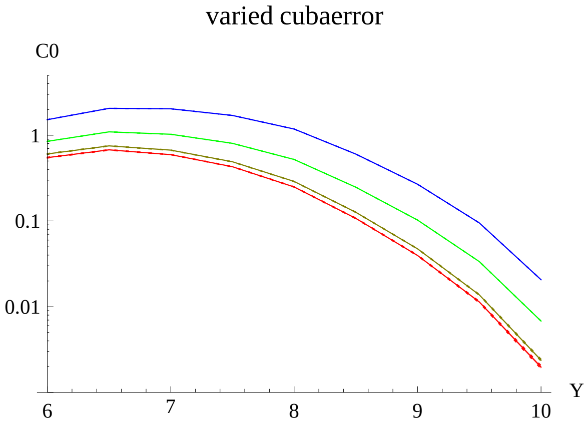


Figure: Differential cross section in dependence on Y for $|\mathbf{k}_{J,1}| = |\mathbf{k}_{J,2}| = 35\,\text{GeV}$. Blue shows the pure LL result, brown the pure NLL result, green the combination of LL vertices with the collinear improved NLL Green's function, red the full NLL vertices with the collinear improved NLL Green's function.

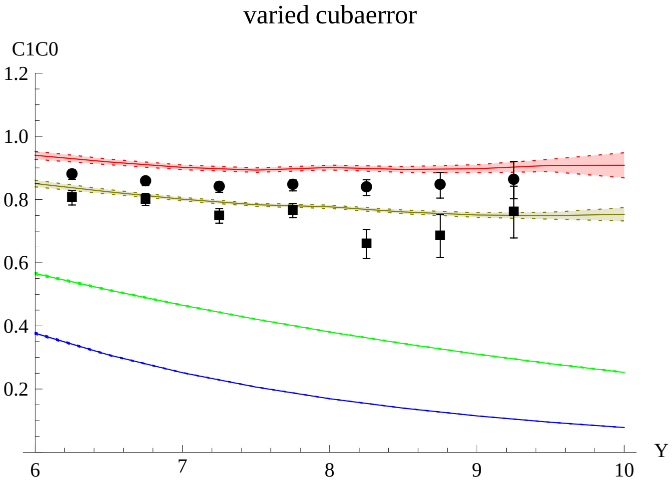


Figure: $\langle \cos \varphi \rangle$ in dependence on Y for $|\mathbf{k}_{J,1}| = |\mathbf{k}_{J,2}| = 35\,\mathrm{GeV}$. As dots are shown the results of S. Cerci and D. d'Enterria obtained with PYTHIA and as squares are shown the results obtained with HERWIG.

comments:

- Parameter $\frac{\alpha_s(Q)N_c}{\pi}\ln\frac{s}{Q^2}$ — "mean number" of undetected gluons radiated in the final state.
  Or tells how many terms of BFKL series are important for convergence
- for the plots just shown: $\frac{\alpha_s N_c}{\pi}\ln\frac{s}{Q^2} \sim 0.85 \div 1.5$.
  Only $1 \div 1.5$ of gluons are produced!
- NLO DGLAP cross section $= D + \alpha_s[A\ln\frac{s}{Q^2} + B] + \mathcal{O}(\frac{Q^2}{s})$
  NLA BFKL series "truncated" just to a single gluon radiation
  $= D + \alpha_s[A\ln\frac{s}{Q^2} + B]$
  where B comes from NLA correction to the vertex
  NLA correction to the kernel gives $\sim \alpha_s^2 \ln\frac{s}{Q^2}$ – equivalent to NNLO DGLAP term
- Take "truncated" BFKL and check fast complicated NLO DGLAP calculations!

Jet vertex in small cone approximation

the (ν, n)-representation for LLA vertex:

$$\frac{\pi\sqrt{2}\,\vec{k}^{\,2}}{\mathcal{C}}\frac{d\Phi^J(\nu,n)}{d\alpha d^2\vec{k}} = \left(\frac{C_A}{C_F}f_g(\alpha) + \sum_{a=q,\bar{q}} f_a(\alpha)\right)\left(\vec{k}^{\,2}\right)^{\gamma} e^{in\phi}\;.$$

Collinear singularities which appear in the NLA calculation are removed by the renormalization of PDFs. The relations between the bare and renormalized quantities are

$$f_q(x) = f_q(x,\mu_F) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\int\limits_x^1 \frac{dz}{z}\left[P_{qq}(z)f_q(\tfrac{x}{z},\mu_F) + P_{qg}(z)f_g(\tfrac{x}{z},\mu_F)\right]\;,$$

$$f_g(x) = f_g(x,\mu_F) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\int\limits_x^1 \frac{dz}{z}\left[P_{gq}(z)f_q(\tfrac{x}{z},\mu_F) + P_{gg}(z)f_g(\tfrac{x}{z},\mu_F)\right]\;,$$

where $\frac{1}{\hat{\epsilon}} = \frac{1}{\epsilon} + \gamma_E - \ln(4\pi) \approx \frac{\Gamma(1-\epsilon)}{\epsilon(4\pi)^{\epsilon}}$

the DGLAP kernels are given by

$$
\begin{aligned}
P_{gq}(z) &= C_F \frac{1+(1-z)^2}{z} \,, \\
P_{qg}(z) &= T_R \left[z^2+(1-z)^2\right] \,, \\
P_{qq}(z) &= C_F \left(\frac{1+z^2}{1-z}\right)_+ = C_F \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right] \,, \\
P_{gg}(z) &= 2C_A \left[\frac{1}{(1-z)_+} + \frac{1}{z} - 2 + z(1-z)\right] + \left(\frac{11}{6}C_A - \frac{n_f}{3}\right)\delta(1-z) \,,
\end{aligned}
$$

with $T_R = 1/2$. We adopt the $\overline{\mathrm{MS}}$ scheme.
the QCD charge renormalization,

$$
\alpha_s = \alpha_s(\mu_R)\left[1 + \frac{\alpha_s(\mu_R)}{4\pi}\beta_0\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu_R^2}{\mu^2}\right)\right] \,, \quad \beta_0 = \frac{11C_A}{3} - \frac{2n_f}{3} \,,
$$

jet cone:

$$\Delta\phi^2 + \Delta y^2 \leq R^2$$

At NLA we have both the virtual corrections and two-particle production in the parton-Reggeon collision. The jet in the latter case can be either produced by one of the two partons or by both together. If we call the produced partons a and b, we have the following contributions:

1. the parton a generates the jet, while the parton b can have arbitrary kinematics, provided that it lies *outside* the jet cone;
2. similarly with $a \leftrightarrow b$;
3. the two partons a and b both generate the jet.

- gluon radiation down from the proton fragmentation to the central rapidity region has to be subtracted.
  BFKL counterterm.

at $R \to 0$

- allows to get analytic result for jet vertex projected on $|n, \nu\rangle$. Easy numerics for observables.
- Furman(1981): the dependence on the cone size is of the form $A \ln R + B + \mathcal{O}(R^2)$.
- Jager, Stratmann, Vogelsang: at $s \sim Q^2$, very good agreement between the results of the small-cone approximation and the Monte Carlo calculations, even for cone sizes of up to $R = 0.7$

$$
I_q = \frac{\alpha_s}{2\pi}\left(\vec{k}^{\,2}\right)^{\gamma-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\,\right)^{n}\int\limits_{\alpha}^{1}\frac{d\zeta}{\zeta}\sum_{a=q,\bar{q}} f_a\left(\frac{\alpha}{\zeta}\right)
$$

$$
\times\left[\left\{P_{qq}(\zeta)+\frac{C_A}{C_F}P_{gq}(\zeta)\right\}\ln\frac{\vec{k}^{\,2}}{\mu_F^2}-2\zeta^{-2\gamma}\ln R\,\{P_{qq}(\zeta)+P_{gq}(\zeta)\}-\frac{\beta_0}{2}\ln\frac{\vec{k}^{\,2}}{\mu_R^2}\delta(1-\zeta)\right.
$$

$$
+C_A\delta(1-\zeta)\left\{\chi(n,\gamma)\ln\frac{s_0}{\vec{k}^{\,2}}+\frac{85}{18}+\frac{\pi^2}{2}+\frac{1}{2}\left(\psi'\left(1+\gamma+\frac{n}{2}\right)-\psi'\left(\frac{n}{2}-\gamma\right)-\chi^2(n,\gamma)\right)\right\}
$$

$$
+(1+\zeta^2)\left\{C_A\left(\frac{(1+\zeta^{-2\gamma})\,\chi(n,\gamma)}{2(1-\zeta)_+}-\zeta^{-2\gamma}\left(\frac{\ln(1-\zeta)}{1-\zeta}\right)_+\right)+\left(C_F-\frac{C_A}{2}\right)\left[\frac{\bar{\zeta}}{\zeta^2}I_2-\frac{2\ln}{1-}\right.\right.
$$

$$
\left.\left.+2\left(\frac{\ln(1-\zeta)}{1-\zeta}\right)_+\right]\right\}+\delta(1-\zeta)\left(C_F\left(3\ln 2-\frac{\pi^2}{3}-\frac{9}{2}\right)-\frac{5n_f}{9}\right)
$$

$$
\left.+C_A\zeta+C_F\bar{\zeta}+\frac{1+\bar{\zeta}^2}{\zeta}\left\{C_A\frac{\bar{\zeta}}{\zeta}I_1+2C_A\ln\frac{\bar{\zeta}}{\zeta}+C_F\zeta^{-2\gamma}(\chi(n,\gamma)-2\ln\bar{\zeta})\right\}\right]\ .
$$

$\gamma = i\nu - \frac{1}{2}$

$$I_g = \frac{\alpha_s}{2\pi}\left(\vec{k}^{\,2}\right)^{\gamma-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\,\right)^n \int\limits_\alpha^1 \frac{d\zeta}{\zeta} f_g\left(\frac{\alpha}{\zeta}\right)\frac{C_A}{C_F}$$

$$\times\left\{\left\{P_{gg}(\zeta)+2\,n_f\frac{C_F}{C_A}P_{qg}(\zeta)\right\}\ln\frac{\vec{k}^{\,2}}{\mu_F^2}-2\zeta^{-2\gamma}\ln R\left\{P_{gg}(\zeta)+2\,n_f P_{qg}(\zeta)\right\}\right.$$

$$+C_A\delta(1-\zeta)\left\{\chi(n,\gamma)\ln\frac{s_0}{\vec{k}^{\,2}}+\frac{1}{12}+\frac{\pi^2}{6}+\frac{1}{2}\left(\psi'\left(1+\gamma+\frac{n}{2}\right)-\psi'\left(\frac{n}{2}-\gamma\right)-\chi^2(n,\gamma)\right)\right\}$$

$$-\frac{\beta_0}{2}\ln\frac{\vec{k}^{\,2}}{4\mu_R^2}\delta(1-\zeta)+\,2C_A(1-\zeta^{-2\gamma})\left(\left(\frac{1}{\zeta}-2+\zeta\bar{\zeta}\right)\ln\bar{\zeta}+\left(\frac{\ln(1-\zeta)}{1-\zeta}\right)_+\right)$$

$$+C_A\,\left[\frac{1}{\zeta}+\frac{1}{(1-\zeta)_+}-2+\zeta\bar{\zeta}\right]\left((1+\zeta^{-2\gamma})\chi(n,\gamma)-2\ln\zeta+\frac{\bar{\zeta}^2}{\zeta^2}I_2\right)$$

$$\left.+n_f\left[2\zeta\bar{\zeta}\,\frac{C_F}{C_A}+(\zeta^2+\bar{\zeta}^2)\left(\frac{C_F}{C_A}\chi(n,\gamma)+\frac{\bar{\zeta}}{\zeta}I_3\right)-\frac{1}{12}\delta(1-\zeta)\right]\right\}\,.$$

$I_{1,2,3}$ functions:

$$I_2 = \frac{\zeta^2}{\bar\zeta^2}\left[\zeta\left(\frac{{}_2F_1(1,1+\gamma-\frac{n}{2},2+\gamma-\frac{n}{2},\zeta)}{\frac{n}{2}-\gamma-1} - \frac{{}_2F_1(1,1+\gamma+\frac{n}{2},2+\gamma+\frac{n}{2},\zeta)}{\frac{n}{2}+\gamma+1}\right)\right.$$

$$+\zeta^{-2\gamma}\left(\frac{{}_2F_1(1,-\gamma-\frac{n}{2},1-\gamma-\frac{n}{2},\zeta)}{\frac{n}{2}+\gamma} - \frac{{}_2F_1(1,-\gamma+\frac{n}{2},1-\gamma+\frac{n}{2},\zeta)}{\frac{n}{2}-\gamma}\right)$$

$$\left.+\left(1+\zeta^{-2\gamma}\right)\left(\chi(n,\gamma)-2\ln\bar\zeta\right)+2\ln\zeta\right]\ ,$$

$$I_1 = \frac{\bar\zeta}{2\zeta}I_2 + \frac{\zeta}{\bar\zeta}\left[\ln\zeta + \frac{1-\zeta^{-2\gamma}}{2}\left(\chi(n,\gamma)-2\ln\bar\zeta\right)\right]\ ,$$

$$I_3 = \frac{\bar\zeta}{2\zeta}I_2 - \frac{\zeta}{\bar\zeta}\left[\ln\zeta + \frac{1-\zeta^{-2\gamma}}{2}\left(\chi(n,\gamma)-2\ln\bar\zeta\right)\right]\ .$$

property of the hypergeometric function,

$${}_2F_1(1,a,a+1,\zeta) = a(\psi(1)-\psi(a)-\ln\bar\zeta) + \mathcal{O}(\bar\zeta\ln\bar\zeta)\ ,$$

one can easily see that for $\zeta \to 1$,

$$I_2 = \mathcal{O}\left(\frac{\ln\bar\zeta}{\bar\zeta}\right)\ , \quad I_1 = \mathcal{O}(\ln\bar\zeta)\ , \quad I_3 = \mathcal{O}(\ln\bar\zeta)\ ,$$

which implies that the integral over ζ is convergent on the upper limit.

Related processes

- forward jets electroproduction $\gamma^* p \to X + Jet$ [Lidia's talk]
  NLA $\gamma^* \to \gamma^*$ IF is known.
- processes with identified hadrons:
  $\gamma^* p \to X + h$, (H1 measurement of a forward hard pion electroproduction)
  $pp \to h_1 + X + h_2$ – analog of Mueller-Navelet jets process

We derived NLA vertex for these processes. It is expressed in terms of collinear fragmentation functions of detected hadron.

$$\vec{k}_h^{\,2}\,\frac{d\Phi^h(\nu,n)}{d\alpha_h d^2\vec{k}_h} = 2\,\alpha_s(\mu_R)\sqrt{\frac{C_F}{C_A}}\,\left(\vec{k}_h^{\,2}\right)^{\gamma-\frac{n}{2}}\left(\vec{k}_h\cdot\vec{l}\,\right)^n$$

$$\times\left\{\int\limits_{\alpha_h}^{1}\frac{dx}{x}\left(\frac{x}{\alpha_h}\right)^{2\gamma}\left[\frac{C_A}{C_F}f_g(x)D_g^h\left(\frac{\alpha_h}{x}\right)+\sum_{i=q,\bar{q}}f_{q_i}(x)D_{q_i}^h\left(\frac{\alpha_h}{x}\right)\right]\right.$$

$$+\frac{\alpha_s\left(\mu_R\right)}{2\pi}\int\limits_{\alpha_h}^{1}\frac{dx}{x}\int\limits_{\frac{\alpha_h}{x}}^{1}\frac{d\zeta}{\zeta}\left(\frac{x\,\zeta}{\alpha_h}\right)^{2\gamma}\left[\frac{C_A}{C_F}f_g(x)D_g^h\left(\frac{\alpha_h}{x\zeta}\right)C_{gg}\left(x,\zeta\right)\right.$$

$$+\sum_{i=q,\bar{q}}f_{q_i}(x)D_{q_i}^h\left(\frac{\alpha_h}{x\zeta}\right)C_{qq}\left(x,\zeta\right)+\sum_{i=q,\bar{q}}f_{q_i}(x)D_g^h\left(\frac{\alpha_h}{x\zeta}\right)C_{qg}\left(x,\zeta\right)$$

$$\left.\left.+\frac{C_A}{C_F}f_g(x)\sum_{i=q,\bar{q}}D_{q_i}^h\left(\frac{\alpha_h}{x\zeta}\right)C_{gq}\left(x,\zeta\right)\right]\right\}$$

NLA coefficient functions:

$$C_{gg}\left(x,\zeta\right)=P_{gg}(\zeta)\left(1+\zeta^{-2\gamma}\right)\ln\left(\frac{\vec{k}_h^2x^2\zeta^2}{\mu_F^2\alpha_h^2}\right)-\frac{\beta_0}{2}\ln\left(\frac{\vec{k}_h^2x^2\zeta^2}{\mu_R^2\alpha_h^2}\right)$$

$$+\,\delta(1-\zeta)\left[C_A\ln\left(\frac{s_0\,\alpha_h^2}{\vec{k}_h^2\,x^2}\right)\chi(n,\gamma)-C_A\left(\frac{67}{18}-\frac{\pi^2}{2}\right)+\frac{5}{9}n_f\right.$$

$$\left.+\frac{C_A}{2}\left(\psi'\left(1+\gamma+\frac{n}{2}\right)-\psi'\left(\frac{n}{2}-\gamma\right)-\chi^2(n,\gamma)\right)\right)\Bigg]$$

$$+\,C_A\left(\frac{1}{\zeta}+\frac{1}{(1-\zeta)_+}-2+\zeta\bar{\zeta}\right)$$

$$\times\left(\chi(n,\gamma)(1+\zeta^{-2\gamma})-2(1+2\zeta^{-2\gamma})\ln\zeta+\frac{\bar{\zeta}^2}{\zeta^2}I_2\right)$$

$$+\,2\,C_A(1+\zeta^{-2\gamma})\left(\left(\frac{1}{\zeta}-2+\zeta\bar{\zeta}\right)\ln\bar{\zeta}+\left(\frac{\ln(1-\zeta)}{1-z}\right)_+\right)\ ,$$

- Parateter $\eta = \frac{N_c \alpha_s(p_t)}{\pi} \Delta y$ gives an idea how many hard undetected gluons are radiated.
  Regime: $\eta \sim 1$ and $\alpha_s(p_t)$ – small:
  BFKL and fixed order DGLAP should give similar results
  truncated BFKL – simple way to control complicated (numerical) DGLAP calculations.
- Analitic results for forward identified hadron and small cone jet IFs in $|n, \nu\rangle$ representation are derived.
- Numerical implementation for MuellerNavelet process is in progress.

Summary and prospects

More interesting regime:
$\eta = \frac{N_c \alpha_s(p_t)}{\pi} \Delta y \sim 2 \div 3$ and $\alpha_s(p_t)$ – small
experiment: go to smaller p_t (larger α_s)?

Prospects (what can be predicted):

- electroproduction:
  production of forward Jet and identified hadron h
  $\gamma^* p \to X + Jet$ and $\gamma^* p \to X + h$
- proton proton collisions:
  production of a pair of rapidity separated identified hadrons, h_1, h_2
  $p\, p \to h_1 + X + h_2$